UNIT-I

Per Unit System of Representation

In a large interconnected power system with various voltage levels, and various capacity equipments it is convenient to work with per unit (p.u.) system of quantities for analysis purposes rather than in absolute values of quantities.

The per unit value (p.u.) of any quantity is defined as

$$\text{p.u. Value} = \frac{\text{Actual value of the quantity (in any unit)}}{\text{The base (or) reference value in the same unit}}$$

The ratings of the equipments in a power system are given in terms of operating voltage and the capacity in KVA. Therefore, it is found convenient and useful to select voltage and KVA as the base quantities.

Let 'V_b' be the base voltage and KVA_b be the base kilovoltamperes, then

$$V_{p.u.} = \frac{V_{actual}}{V_b}$$

$$\text{Base Current} = \frac{KVA_b \times 1000}{V_b}$$

$$\therefore \text{p.u. Current} = \frac{\text{Actual Current}}{\text{Base Current}} = \frac{\text{Actual Current}}{KVA_b \times 1000} \times V_b$$

$$\text{Base impedance} = \frac{\text{Base voltage}}{\text{Base Current}} = \frac{(V_b)^2}{KVA_b \times 1000}$$

$$\therefore \text{p.u. impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}} = \frac{Z \cdot KVA_b \times 1000}{(V_b)^2}$$

$$= \frac{Z\,(MVA)_b}{(V_b)^2}$$

From above equation, it is clear that the p.u. impedance is directly proportional to base kVA and inversely proportional to square of base voltage.

It is desired to find the p.u. impedance of various equipments corresponding to the common base voltage and kVA. If the individual quantities are $Z_{pu,old}$, kVA_{old} and V_{old} and the common base quantities are $Z_{p.u.,new}$, kVA_{new} and V_{new}, then making use of the above relation,

$$Z_{p.u.,new} = Z_{p.u.,old} \times \frac{kVA_{new}}{kVA_{old}} \times \left(\frac{V_{old}}{V_{new}}\right)^2$$

The p.u. impedance of an equipment corresponding to its own rating is given by

$$Z_{pu} = \frac{I.Z}{V}$$

where, Z = Absolute value of impedance of the equipment.

Let the impedance of transformer referred to primary side be Z_p and that on the secondary side be Z_s, then

$$Z_p = Z_s \cdot \left(\frac{V_p}{V_s}\right)^2$$

where V_p and V_s are the primary and secondary voltages of the transformer

Now, $$Z_{p,p.u} = \frac{Z_p \cdot I_p}{V_p} = Z_s \cdot \left(\frac{V_p}{V_s}\right)^2 \left(\frac{I_p}{V_p}\right)$$

$$= Z_s \cdot \frac{V_p I_p}{V_s^2} = Z_s \cdot \frac{V_s \cdot I_s}{V_s^2} = \frac{Z_s \cdot I_s}{V_s}$$

$$= Z_{s,pu}.$$

Hence, the p.u. impedence of transformer referred to primary side is equal to the p.u. impedance of transformer referred to the secondary side.

The different voltage levels in a power system are due to ~~the~~ the presence of transformers. Therefore, the procedure for selecting base voltage is as follows:

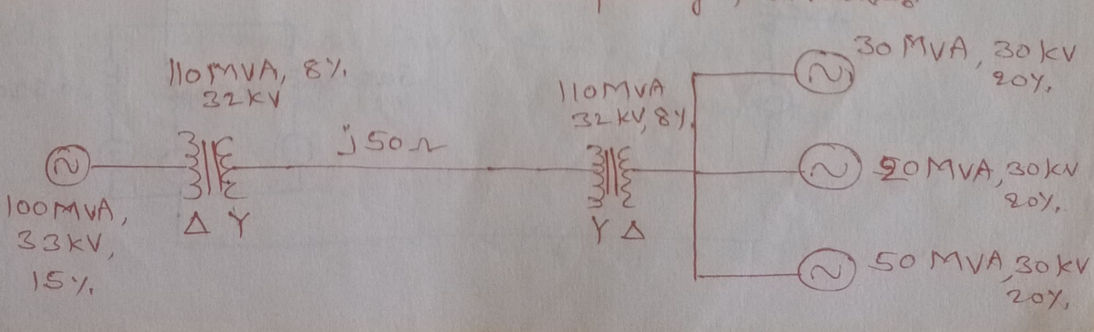
A voltage corresponding to any part of the system could be taken as a base and the base voltages in other parts of the circuit, separated from the original part by transformers is related through the turns ratio of the transformers.

If the base voltage on primary side is V_{pb} then on the secondary side of the transformer, the base voltage will be

$$V_{sb} = V_{pb}\,(N_s/N_p)$$

where N_s and N_p are turns of transformer on secondary and primary respectively.

Ex:- A 100MVA, 33kV 3-phase generator has a subtransient reactance of 15%. The generator is connected to the motors through ~~the~~ a transmission line and transformers as shown in the following figure. The motors have rated inputs of 30 MVA, 20MVA and 50MVA at 30kV with 20% subtransient reactance. The 3-phase transformers are rated at 110MVA, 32 kV, Δ/110kV Y with leakage reactance of 8%. The line has a reactance of 50Ω. Selecting the generator rating as the base quantities in the generator circuit, determine the base quantities in other parts of the system & evaluate the corresponding p.u. values.



Sol:-

Assuming base values as 100 MVA and 33 kV in the generator circuit, the p.u. reactance of generator will ... The base value of voltage in the line will be

$$33 \times \frac{110}{32} = 113.43\text{ kV}$$

In the motor circuit,

$$113.43 \times \frac{32}{110} = 33\text{ kV}$$

The reactance of the transformer given is 8% corresponding to 110 MVA, 32 kV. Therefore, corresponding to 100 MVA and 33 kV the p.u. reactance will be

$$0.08 \times \left(\frac{100}{110}\right) \times \left(\frac{32}{33}\right)^2 = 0.06838 \text{ p.u}$$

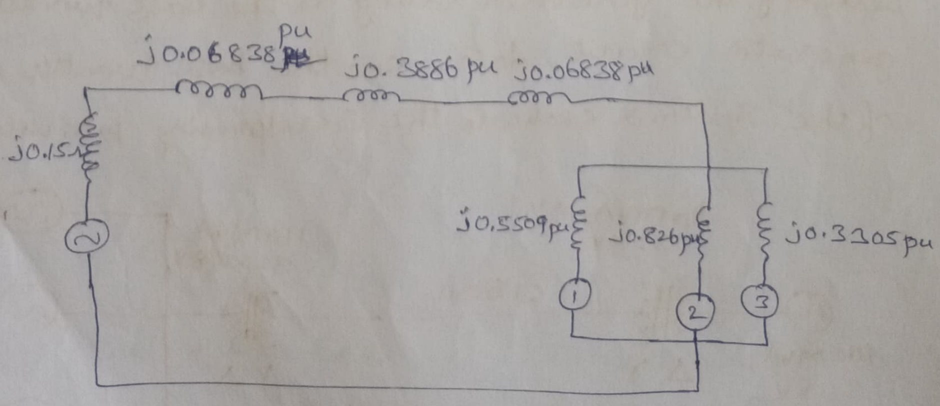
The p.u. impedance of line $= \dfrac{50 \times 100}{(113.43)^2} = 0.3886$ pu.

The p.u reactance of motor 1 $= 0.2 \times \left(\frac{100}{30}\right) \times \left(\frac{30}{33}\right)^2 = 0.5509$ pu

" " " " " 2 $= 0.2 \times \left(\frac{100}{20}\right) \times \left(\frac{30}{33}\right)^2 = 0.826$ pu

" " " " " 3 $= 0.2 \times \left(\frac{100}{50}\right) \times \left(\frac{30}{33}\right)^2 = 0.3305$ pu.

The reactance diagram for the system is shown below.

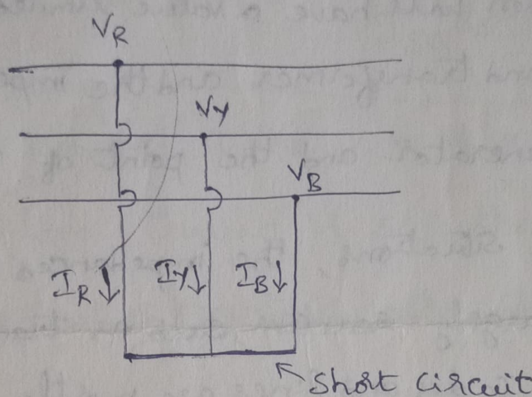


Symmetrical Faults Analysis

Symmetrical faults on 3-phase System

The fault on the power system which gives rise to Symmetrical Currents (i.e. equal fault currents in the lines with 120° displacement) is called a "Symmetrical fault."

The symmetrical fault occurs when all the three conductors of a 3-phase line are brought together simultaneously into a short circuit condition as shown below.



Symmetrical fault

This type of fault gives rise to symmetrical currents i.e. equal fault currents with 120° displacement.

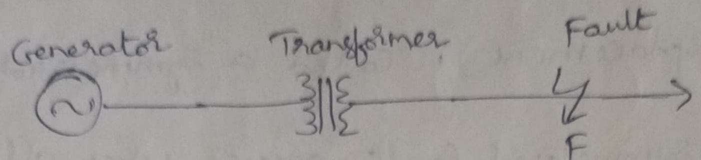
Thus the currents in above figure i.e. I_R, I_Y and I_B will be equal in magnitude with 120° displacement among them. Because of balanced nature of fault, only one phase need to be considered in calculations.

Note:- (a) Symmetrical fault occurs rarely in practice as majority of faults are of unsymmetrical nature.

(b) Symmetrical fault is the most severe and impose more heavy duty on the circuit breaker.

Limitation of fault Current

When short circuit occurs at any point in a sy... the short circuit current is limited by the impedance of system upto the point of fault.



Referring to above figure, if a fault occurs on the feeder at point F, then the short circuit current from the generating station will have a value limited by the impedance of generator and transformer and the impedance of the line between the generator and the point of fault.

In many situations, the impedances limiting the fault current are largely reactive, such as transformers, reactors and generators. Cables and lines are mostly resistive, but the total reactance in calculations exceeds 3 times the resistance hence the resistance upto fault point is neglected.

Percent reactance

The reactance of generators, transformers, reactors etc. is usually expressed in percentage (or) perunit reactance to have simplified calculations.

Percentage (or) perunit reactance is the percentage of total phase voltage dropped in the circuit when full-load current is flowing. i.e.

$$\% X = \left(\frac{I}{V} X\right) \times 100 \quad \text{—— (1)}$$

$I \rightarrow$ full load current, $V \rightarrow$ phase voltage

$X \rightarrow$ reactance in ohms/phase

The percentage (or) per unit reactance can also be expressed in terms of kVA and kV as:

$$\% X = \frac{(kVA) \cdot X}{10\,(kV)^2} \quad \text{—— (2)}$$

where $X \rightarrow$ reactance in ohms

If 'X' is the only reactance in the circuit, then

**
$$\boxed{\begin{aligned} \text{Short circuit current, } I_{SC} &= \frac{V}{X} \\ I_{SC} &= I \times \left(\frac{100}{\% X}\right) \quad [\text{From } ①] \end{aligned}}$$

For example, if the % X of an element is 20% and the full load current is 50A, then $I_{SC} = 50 \times \left(\frac{100}{20}\right) = 250A.$ when only that element is in the circuit.

Short Circuit kVA

Although the potential at the point of fault is zero, it is a normal practice to express the short circuit current in terms of short circuit kVA based on the normal system voltage at the point of fault.

The product of normal system voltage and short circuit current at the point of fault expressed in kVA is known as 'Short circuit kVA.'

Let $V \rightarrow$ Normal phase voltage, $I \rightarrow$ full load current in Amperes at base kVA

$\% X \rightarrow$ % reactance of the system on base kVA upto the fault point

$$I_{SC} = I\left(\frac{100}{\% X}\right)$$

** ∴ Short Circuit kVA for 3-phase Circuit

$$= \frac{3 V I_{sc}}{1000}$$

$$= \frac{3 . V I}{1000} \times \frac{100}{\% X}$$

$$(kVA)_{sc} = (\text{Base kVA}) \times \frac{100}{\% X}$$

Application of Series reactors for Control of Short circuit Currents

When the power system expands, the faults level also rises. The circuit breakers connected must be capable of dealing with maximum possible short-circuit currents that can occur at their points of connection.

Generally, the reactance of the system under fault conditions is low and fault currents may rise to a dangerously high value. If no steps are taken to limit the value of these short circuit currents, circuit breakers ~~are~~ have heavy duty and they can damage the lines & other equipments.

To limit the short circuit current to a value that can be handled by the circuit breakers, additional reactances known as "series reactors" are connected in series with the system at suitable points.

Advantages of reactors

(i) Limits the short circuit current flow & hence protects the equipment from overheating

(ii) Troubles are localised (or) isolated at the point where they originate without communicating their disturbing effects to other parts of the power system.

(iii) They permit the installation of circuit breakers of lower rating.

Location of Reactors

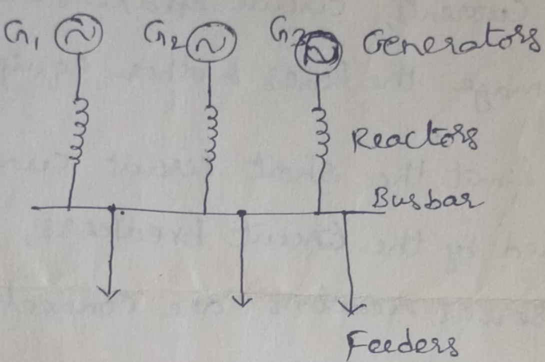
Short circuit current limiting reactors may be connected (i) in series with each generator,

(ii) in series with each feeder and

(iii) in bus bars

(1) Generator Reactors

When the reactors are connected in series with the generator, they are known as "generator reactors".



In this case, reactor may be considered as a part of leakage reactance of the generator, hence its effect is to protect the generator in the case of any short circuit beyond the reactors.

Disadvantages

(i) There is a constant voltage drop and power loss in the reactors even during the normal operation.

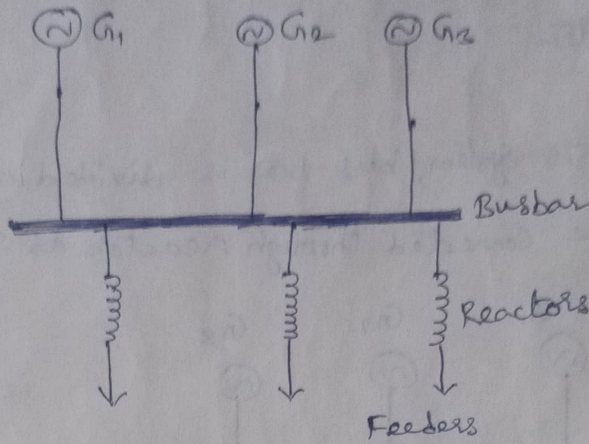
(ii) If a busbar (or) feeder fault occurs close to the busbar, the voltage at the busbar will be reduced to a low value, thereby causing the generators fall out of step.

(iii) If a fault occurs on any feeder, the continuity of supply to others is likely to be affected.

Hence it is not a common practice to use separate reactors for generators.

(2) Feeder Reactors

When the reactors are connected in series with each feeder, they are known as feeder reactors.



Since most of the short circuits occur on feeders, a large number of reactors are used for such circuits.

Advantages

1. If a fault occurs on any feeder, the voltage drop in its reactor will not affect the busbar voltage so that there is a little tendency for the generator to lose synchronism.
2. The fault on a feeder will not affect other feeders and consequently the effects of faults are localised.

Disadvantages

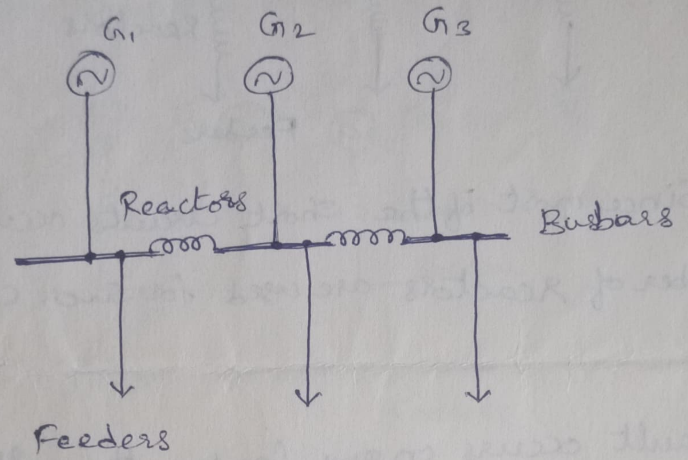
1. There is a constant power loss and voltage drop in the reactors even during normal operation.
2. If a short circuit occurs at the busbars, no protection is provided to the generators.
3. If the no. of generators is increased, the sizes of feeder reactors will have to be increased to keep the short circuit currents within the ratings of the feeder circuit breakers.

(3) Busbar Reactors

The disadvantages of the above two Schemes Can be overcome by locating the reactors in the bus-bars. There are two methods for this purpose, namely: Ring System and Tie-bar system

(i) Ring system

In this system, bus-bar is divided into sections and these sections are Connected through reactors as shown below.



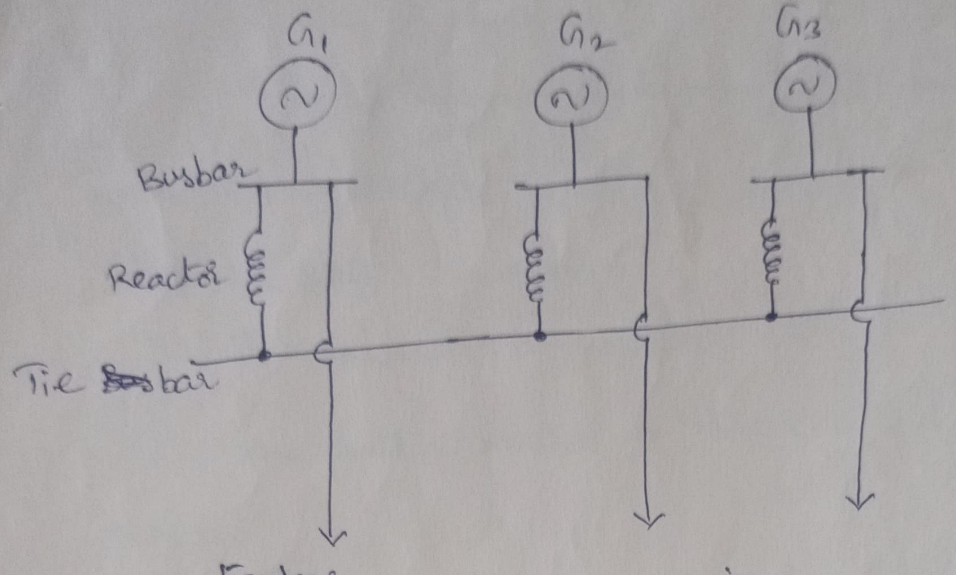
Generally, one feeder is fed from one generator only. Under normal operating conditions, each generator will supply its own section of the load and very little power will be fed by other generators. This results in low power loss and voltage drop in the reactors.

However, the principal advantage of the system is that if a fault occurs on any feeders, only one generator mainly feeds the fault while the currents fed from other generators is small due to the presence of reactors.

Therefore, only that section of busbar is affected to which the feeder is connected, the other sections being able to continue in normal operation.

(ii) Tie bar System

Following figure shows the tie bar system.



Compared to the ring system, in tie bar system between each section there are two series reactors so that reactors must have half the reactance of those used in the ring system.

Another advantage of tie bar system is that additional generators may be connected to the system without requiring changes in the existing reactors.

However, this system has the disadvantage that it requires an additional busbar i.e. tie bar.

Symmetrical Component Theory

The Solution of unsymmetrical fault problems can be obtained by either

(a) kirchoff's laws (or)

(b) Symmetrical Components method.

The latter method is preferred because of the following reasons.

(i) It is a simple method and gives more generality to be given to fault performance studies

(ii) It provides a useful tool for the protection engineers, particularly in Connection with tracing out of fault Currents.

Symmetrical Component Method

Any unbalanced system of 3-phase currents ~~having~~ may be ~~positive~~ regarded as being Composed of three Separate sets of balanced vectors

(i) Positive sequence Currents

(ii) Negative Sequence Currents

(iii) Zero Sequence Currents

The positive, negative and Zero sequence components are called "Symmetrical Components" of the original unbalanced System.

The ~~Symbols~~ Subscripts 1, 2 and 0 are generally used to indicate positive, negative and zero phase sequence

Components respectively.

Ex:- For instance, $\vec{I}_{R0}$ indicates the zero phase sequence Component of the current in the 'R' phase.

Similarly, $\vec{I}_{Y1}$ implies the positive phase sequence component of Current in the 'Y' phase.

Illustration

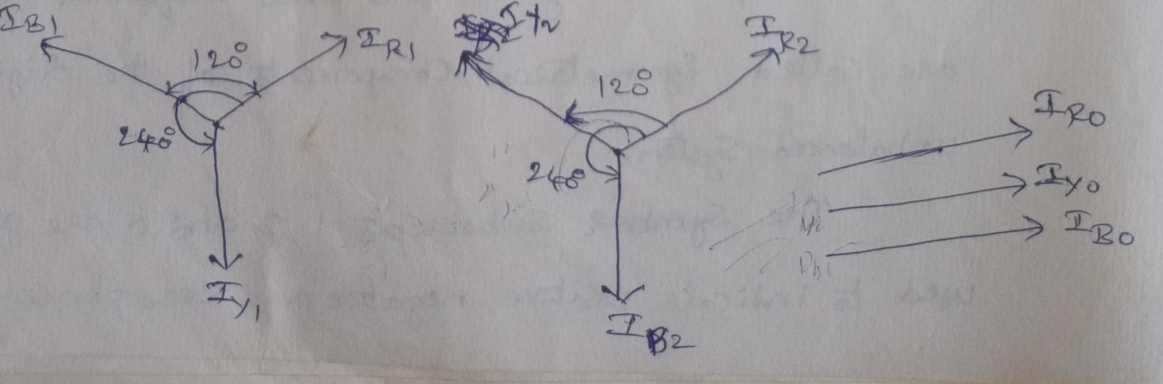
Let us apply the Symmetrical Components theory to an unbalanced 3-phase System.

Suppose an unsymmetrical fault occurs on a 3-phase System having phase Sequence RYB. According to Symmetrical Components theory, the resulting unbalanced Currents $\vec{I}_R$, $\vec{I}_Y$, and $\vec{I}_B$ can be resolved into:

(i) a balanced system of 3-phase Currents $\vec{I}_{R1}$, $\vec{I}_{Y1}$ and $\vec{I}_{B1}$ having positive phase sequence. These are the positive phase Sequence Components.

(ii) a balanced System of 3-phase Currents $\vec{I}_{R2}$, $\vec{I}_{Y2}$ and $\vec{I}_{B2}$ having negative phase sequence. These are negative phase sequence components.

(iii) a system of three Currents $\vec{I}_{R0}$, $\vec{I}_{Y0}$ and $\vec{I}_{B0}$ equal in magnitude with zero phase displacement from each other. These are the zero phase sequence components.



The current in any phase is equal to the vector sum of positive, negative and zero phase sequence currents in that phase.

$$I_R = I_{R_1} + I_{R_2} + I_{R_0}$$

$$I_Y = I_{Y_1} + I_{Y_2} + I_{Y_0}$$

$$I_B = I_{B_1} + I_{B_2} + I_{B_0}$$

Note:-

(i) The positive, negative and zero sequence currents form balanced system of currents. Hence they are called Symmetrical Components of the unbalanced system.

(ii) The symmetrical component theory applies equally to 3-phase currents and voltages both phase and line values.

(iii) The symmetrical components do not have separate existence. They are only mathematical components of unbalanced currents which actually flow in the system.

(iv) In a balanced 3-phase system, negative and zero sequence currents are zero.)

Symmetrical Components in terms of Phase Currents

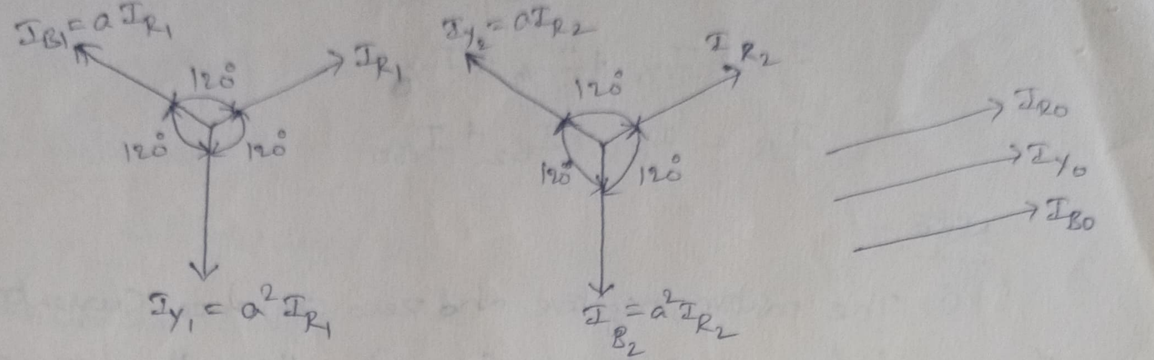
The unbalanced phase currents in a 3-phase system can be expressed in terms of symmetrical components as

$$I_R = \vec{I}_{R_1} + \vec{I}_{R_2} + \vec{I}_{R_0}$$

$$I_Y = \vec{I}_{Y_1} + \vec{I}_{Y_2} + \vec{I}_{Y_0}$$

$$I_B = \vec{I}_{B_1} + \vec{I}_{B_2} + \vec{I}_{B_0}$$

Following figure represents the phasor representation of Symmetrical Components. It is advantageous to express the symmetrical Components in terms of unbalanced fault Currents.



Here operator 'a' is having magnitude 'one' which when multiplied to a vector rotates the vector through 120° in the anticlockwise direction.

$$a = 1 \angle 120° = -0.5 + j0.866$$

$$a^2 = 1 \angle 240° = -0.5 - j0.866$$

$$a^3 = 1 \angle 0° = 1$$

$$1 + a + a^2 = 0$$

It is clear from the above figure that,

$$\vec{I}_R = \vec{I}_{R_1} + \vec{I}_{R_2} + \vec{I}_{R_0} \quad -①$$

$$\vec{I}_Y = \vec{I}_{Y_1} + \vec{I}_{Y_2} + \vec{I}_{Y_0} = a^2 \vec{I}_{R_1} + a\vec{I}_{R_2} + \vec{I}_{R_0} \quad -②$$

$$\vec{I}_B = \vec{I}_{B_1} + \vec{I}_{B_2} + \vec{I}_{B_0} = a\vec{I}_{R_1} + a^2\vec{I}_{R_2} + \vec{I}_{R_0} \quad -③$$

Zero Sequence Current

Adding ①, ② and ③,

$$I_R + I_Y + I_B = 3 I_{R_0} \Rightarrow I_{R_0} = \frac{1}{3}(I_R + I_Y + I_B)$$

Positive sequence Current

Adding ② × a and ③ × a^2 to ①, we get

$$I_R + a I_Y + a^2 I_B = 3 I_{R_1} \Rightarrow I_{R_1} = \frac{1}{3}(I_R + a I_Y + a^2 I_B)$$

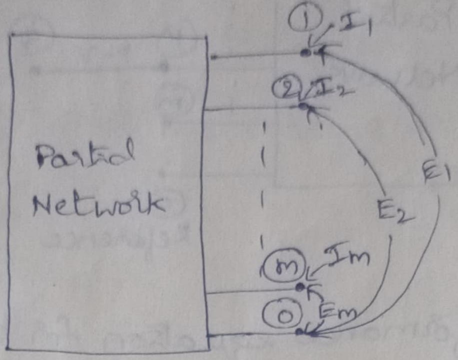
ly Negative Sequence Current, $I_{R_2} = \frac{1}{3}(I_R + a^2 I_Y + a I_B)$

UNIT - II

POWER SYSTEM NETWORK MATRICES

Formation of Z_{BUS} - Partial Network

Assume that the bus impedance matrix Z_{BUS} is known for a partial network of 'm' buses and a reference node 'o'.



The performance equation of this network for above figure is $\bar{E}_{BUS} = Z_{BUS} \bar{I}_{BUS}$

$\bar{E}_{BUS}$ = an m×1 vector of bus voltages measured with respect to the reference node

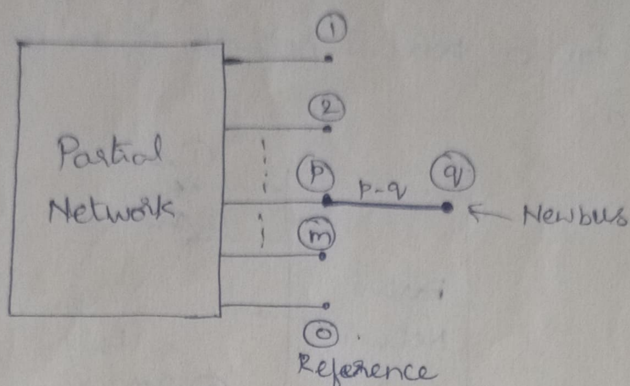
$\bar{I}_{BUS}$ = an m×1 vector of impressed bus currents

If an element p-q is a branch, a new bus 'q' is added to the partial network and the resultant bus impedance matrix is of dimension (m+1) × (m+1). The new voltage & current vectors are of dimension (m+1)×1.

If the element p-q is a link, no new bus is added to the partial network. In this case, the dimensions of the matrices in the performance equation are unchanged; but all the elements of the bus impedance matrix must be calculated to include the effect of the added link.

Addition of element (branch) from new bus to an old bus

When an element p-q is added to the partial network between a new bus to an old bus, the network can be changed as shown below:



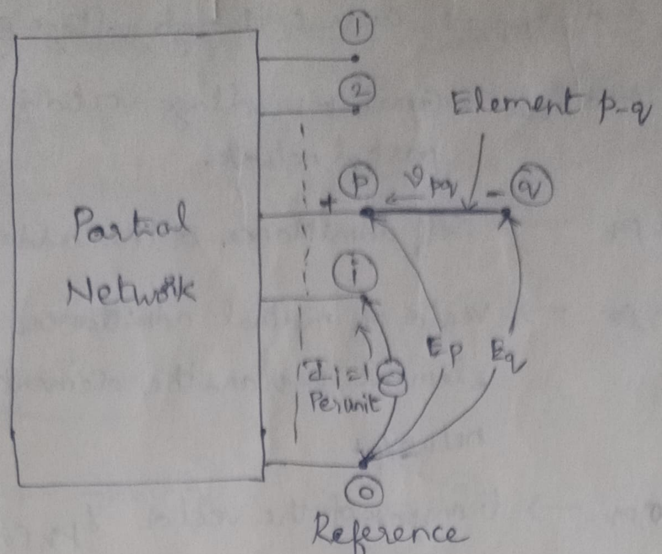
The performance equation for the partial network with an added branch p-q is

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ \vdots \\ E_q \end{bmatrix} = \begin{array}{c|cccccccc} & 1 & 2 & \cdots & p & \cdots & m & q \\ \hline 1 & Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ 2 & Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots \\ p & Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots \\ m & Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ q & Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{array} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \quad \text{— (1)}$$

It is assumed that the network consists of bilateral passive elements. Hence $Z_{qi} = Z_{iq}$ where $i = 1, 2 \ldots m$ and refers to the buses of the partial network, not including the new bus 'q'.

The added branch p-q is assumed to be mutually coupled with one or more elements of the partial network.

The elements Z_{qi} can be determined by injecting a current at the ith bus and calculating the voltage at the q^{th} bus w.r.t. the reference node as shown in following figure.



Since all other bus currents are equal to zero, it follows from equation ① that,

$$\left.\begin{aligned} E_1 &= Z_{1i} I_i \\ E_2 &= Z_{2i} I_i \\ &\cdots \\ E_p &= Z_{pi} I_i \\ &\cdots \\ E_m &= Z_{mi} I_i \\ E_q &= Z_{qi} I_i \end{aligned}\right\} \quad \text{— ②}$$

Let $I_i = 1$ per unit in. eqn ②, Z_{qi} can be obtained directly by calculating E_q.

The bus voltages associated with the added element and the ~~bus~~ voltage across the element are related by

$$E_q = E_p - v_{pq} \quad \text{— ③}$$

The currents in the elements of the network in the above figure are expressed in terms of primitive admittances and voltages across the elements by

$$\begin{bmatrix} i_{pq} \\ i_{\rho\sigma} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & y_{pq,\rho\sigma} \\ y_{\rho\sigma,pq} & y_{\rho\sigma,\rho\sigma} \end{bmatrix} \begin{bmatrix} v_{pq} \\ v_{\rho\sigma} \end{bmatrix} \quad \text{— ④}$$

In equation ④, pq is a fixed subscript and refers to the added element and $\rho\sigma$ is a variable subscript and refers to all other elements.

i_{pq} and v_{pq} → current through, voltage across the added e

$i_{e\sigma}$ and $v_{e\sigma}$ → Current & voltage vectors of the elements of partial network.

$y_{pq\,pq}$ → Self admittance of the added elements

$\bar{y}_{pq,e\sigma}$ → vector of mutual admittances between the added element p-q and the elements ρ-σ of the partial network.

$\bar{y}_{e\sigma,pq}$ → transpose of the vector $\bar{y}_{pq\,e\sigma}$

$[\bar{y}_{e\sigma,e\sigma}]$ → primitive admittance matrix of the partial network.

The current in the added branch is $i_{pq} = 0$ — (5)

However, 'v_{pq}' is not equal to zero since the added branch is mutually coupled to one or more of the elements of the partial network. And also,

$$\bar{v}_{e\sigma} = \bar{E}_{e} - \bar{E}_{\sigma} \quad \text{— (6)}$$

From eqns (4) & (5),

$$i_{pq} = y_{pq,pq}\, v_{pq} + \bar{y}_{pq,e\sigma}\, \bar{v}_{e\sigma} = 0$$

$$\therefore v_{pq} = -\frac{\bar{y}_{pq,e\sigma}\, \bar{v}_{e\sigma}}{y_{pq,pq}}$$

Substituting $\bar{v}_{e\sigma}$ from equation (6), we have

$$v_{pq} = -\frac{\bar{y}_{pq,e\sigma}\,(\bar{E}_{e} - \bar{E}_{\sigma})}{y_{pq,pq}} \quad \text{— (7)}$$

Substituting v_{pq} in eqn (3),

$$E_q = E_p + \frac{\bar{y}_{pq,e\sigma}\,(\bar{E}_{e} - \bar{E}_{\sigma})}{y_{pq,pq}}$$

Finally substituting for E_q, E_p, $\bar{E}_e$ and E_σ from eqn (2) with $I_i = 1$,

$$\boxed{Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,e\sigma}\,(\bar{Z}_{ei} - \bar{Z}_{\sigma i})}{y_{pq,pq}}} \quad i = 1, 2, \ldots m,\ i \neq q \quad \text{— (8)}$$

The element z_{qq} can be calculated by injecting a current at the q^{th} bus and calculating voltage across that bus. Since all other bus currents are zero, it follows from eqn (1) that

$$\left.\begin{aligned} E_1 &= Z_{1q} I_q \\ E_2 &= Z_{2q} I_q \\ &\vdots \\ E_p &= Z_{pq} I_q \\ &\vdots \\ E_m &= Z_{mq} I_q \\ E_q &= Z_{qq} I_q \end{aligned}\right\} \text{ --- (9)}$$

Taking $I_q = 1$ pu, z_{qq} can be obtained directly by calculating E_q.

The voltages at buses p and q are related by eqn (3) and the current through the added element is

$$i_{pq} = -I_q = -1 \text{ --- (10)}$$

From eqns (4) and (10),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma} = -1$$

and therefore,

$$v_{pq} = -\frac{[1 + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}]}{y_{pq,pq}}$$

Substituting $\bar{v}_{\rho\sigma}$ from eqn (6),

$$v_{pq} = -\frac{[1 + \bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)]}{y_{pq,pq}} \text{ --- (11)}$$

Substituting for v_{pq} in eqn (3) from eqn (11),

$$E_q = E_p + \frac{[1 + \bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)]}{y_{pq,pq}}$$

Finally, substituting for E_q, E_p, $\bar{E}_\rho$ & $\bar{E}_\sigma$ from eqn (9), with $I_q = 1$ pu,

$$\boxed{z_{qq} = z_{pq} + \left[\frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{z}_{\rho q} - \bar{z}_{\sigma q})}{y_{pq,pq}}\right]} \text{ --- (12)}$$

(i) If there is no mutual Coupling between the added and other elements of the partial network, then the ele of $\bar{y}_{pq,\rho\sigma}$ are zero. ~~Hence~~ and

$$z_{pq,pq} = \frac{1}{y_{pq,pq}}$$

It follows from equation (8) that

$$z_{qq} = z_{pq} + z_{pq,pq}$$

(ii) Furthermore, if there is no mutual Coupling and 'p' is the reference node,

$$z_{pi} = 0 \quad i = 1, 2, \cdots m \quad i \neq q$$

$$\text{and } z_{qi} = 0 \quad i = 1, 2, \cdots m \quad i \neq q$$

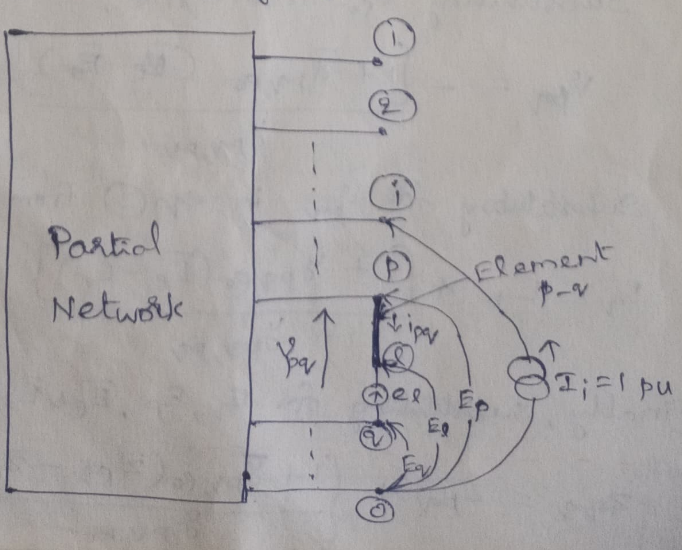
Also

$$z_{pq} = 0 \text{ and therefore,}$$

$$z_{qq} = z_{pq,pq}$$

Addition of a Link (Addition of an element between ~~two~~ old buses)

If the added element p-q is a link, the procedure for recalculating the elements of the bus impedance matrix is to connect in series with the added element a voltage Source e_l as shown in following figure.



This creates a fictitious node 'l' which will be eliminated later. The voltage source 'e_l' is selected such that the current through the added link is zero.

The performance equation for the partial network with the added element p-l and the series voltage source e_l is

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ e_l \end{bmatrix} = \begin{array}{c|c} & \begin{array}{ccccccccc} 1 & 2 & \cdots & p & \cdots & m & l \end{array} \\ \begin{array}{c} 1 \\ 2 \\ \vdots \\ p \\ \vdots \\ m \\ l \end{array} & \left[\begin{array}{cccccc|c} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1l} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2l} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pl} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{ml} \\ \hline Z_{l1} & Z_{l2} & \cdots & Z_{lp} & \cdots & Z_{lm} & Z_{ll} \end{array}\right] \end{array} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ \hline I_l \end{bmatrix} \quad \text{--- (13)}$$

Since $e_l = E_l - E_q$

The element Z_{li} can be determined by injecting a current at the i^{th} bus and calculating the voltage at the l^{th} node w.r.t. bus 'q'. Since all other bus currents are equal to zero, it follows from equation (13) that

$$\left.\begin{aligned} E_k &= Z_{ki} I_i, \quad k = 1, 2, \ldots m \\ e_l &= Z_{li} I_i, \end{aligned}\right\} \text{--- (14)}$$

Letting $I_i = 1$ pu in eqns (14), Z_{li} can be determined by calculating 'e_l'.

The series voltage source is

$$e_l = E_p - E_q - v_{pl} \quad \text{--- (15)}.$$

Since current through the added link is $i_{pq} = 0$, the element p-l can be treated as a branch.

The current in p-l in terms of primitive admittances and the voltage across the elements is

$$i_{pl} = y_{pl,pl}\, v_{pl} + \bar{y}_{pl,\rho\sigma}\, \bar{v}_{\rho\sigma}$$

where, $i_{p\ell} = i_{pq} = 0$

$$\therefore \; v_{p\ell} = - \frac{\bar{y}_{p\ell,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{p\ell,p\ell}}$$

$\because \; \bar{y}_{p\ell,\rho\sigma} = \bar{y}_{pq,\rho\sigma}$ and $y_{p\ell,p\ell} = y_{pq,pq}$

then $$\boxed{v_{p\ell} = - \frac{\bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}}} \quad \text{—— (16)}$$

Substituting in order from eqns (16), (6) and (14) with $I_i = 1$ pu into equation (15) yields

$$\boxed{Z_{\ell i} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}}} \quad \begin{matrix} i = 1,2,\cdots m \\ i \neq \ell \end{matrix} \quad \text{— (17)}$$

- The element $Z_{\ell\ell}$ can be calculated by injecting a current at the ℓ^{th} bus with bus 'q' as reference and calculating the voltage at the ℓ^{th} bus with respect to bus 'q'. Since all other bus currents are equal to zero, it follows from eqn (13) that

$$\left.\begin{aligned} E_k &= Z_{k\ell} I_\ell, \quad k = 1,2,\cdots m \\ e_\ell &= Z_{\ell\ell} I_\ell \end{aligned}\right\} \text{— (18)}$$

Letting $I_\ell = 1$ p.u. in eqn (18), $Z_{\ell\ell}$ can be obtained directly by calculating e_ℓ.

The current in the element p-ℓ is

$$i_{p\ell} = -I_\ell = -1$$

This current in terms of primitive admittances and the voltage across the elements is

$$i_{p\ell} = y_{p\ell,p\ell} v_{p\ell} + \bar{y}_{p\ell,\rho\sigma} \bar{v}_{\rho\sigma} = -1$$

Again, since $\bar{y}_{p\ell,\rho\sigma} = \bar{y}_{pq,\rho\sigma}$ and $y_{p\ell,p\ell} = y_{pq,pq}$

then, $$v_{p\ell} = - \left[\frac{1 + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}}\right] \quad \text{—— (19)}$$

Substituting in order from equations (19), (6) and (18) with $I_l = 1$ pu into (15) yields

$$\boxed{z_{ll} = z_{pl} - z_{ql} + \left[\frac{1 + \bar{y}_{pq,\rho\sigma}\left[\bar{z}_{\rho l} - \bar{z}_{\sigma l}\right]}{y_{pq,pq}}\right]} \quad (20)$$

(i) If there is no mutual coupling between the added element and other elements of the partial network, the elements of $\bar{y}_{pq,\rho\sigma}$ are zero and

$$z_{pq,pq} = \frac{1}{y_{pq,pq}}$$

It follows from eqn (17) that

$$z_{li} = z_{pi} - z_{qi} \qquad \begin{array}{l} i = 1,2,\dots m \\ i \neq l \end{array}$$

and from equation (20),

$$z_{ll} = z_{pl} - z_{ql} + z_{pq,pq}$$

(ii) Furthermore, if there is no mutual coupling and p is the reference node,

$$z_{pi} = 0 \qquad \begin{array}{l} i = 1,2,\dots m \\ i \neq l \end{array}$$

and $z_{li} = -z_{qi}$ $\quad \begin{array}{l} i = 1,2,\dots m \\ i \neq l \end{array}$

Also $z_{pl} = 0$ and therefore,

$$z_{ll} = -z_{ql} + z_{pq,pq}$$

The elements in the l^{th} row & column of Z_{BUS} for the augmented partial network are found from eqn (17) & (20).

Now, it is necessary to calculate the new Z_{BUS} elements to include the effect of added link. It requires modification of the elements z_{ij} where $i, j = 1,2 \dots m$ & eliminating the lth row & column corresponding to the fictitious node.

The node 'l' can be eliminated by short circuiting the series voltage source e_l.

From eqn (13),

$$\bar{E}_{BUS} = Z_{BUS}\bar{I}_{BUS} + \bar{Z}_{il} I_l \quad \text{---} (21)$$

and

$$e_l = \bar{Z}_{lj}\bar{I}_{BUS} + Z_{ll} I_l = 0 \quad \text{---} (22)$$

where $i,j = 1, 2, \ldots m$. solving for I_l from eqn (22) and Substit into (21),

$$\bar{E}_{BUS} = \left[Z_{BUS} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}}\right]\bar{I}_{BUS}$$

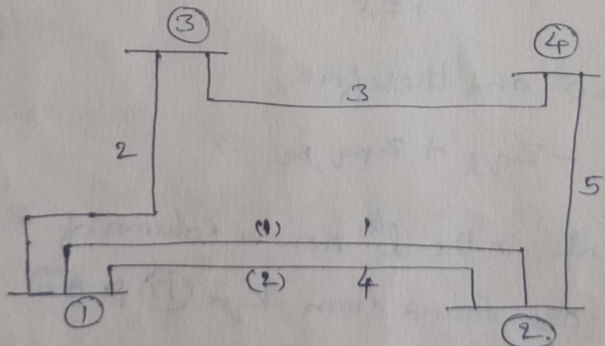
which is the performance equation of the partial network including the link p-q. It follows that the required Z_{BUS} is

$$\boxed{Z_{BUS\,(\text{modified})} = Z_{BUS\,(\text{before elimination})} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}}}$$

where any element of $Z_{BUS\,(\text{modified})}$ is

$$\boxed{Z_{ij\,(\text{modified})} = Z_{ij\,(\text{before elimination})} - \frac{Z_{il}Z_{lj}}{Z_{ll}}}$$

Ex1- (a) Form the bus impedance matrix Z_{BUS} of the network shown in following figure.



(b) Modify the bus impedance matrix obtained in (a) to include the addition of an element from bus 2 to bus 4 with an impedance of 0.3 and coupled to element 5 with a mutual impedance of 0.1.

(c) Modify the bus impedance matrix obtained in part (b) to remove the new element from bus 2 to bus 4.

Element number	Self		Mutual	
	Bus Code $p-q$	Impedance $z_{pq,pq}$	Bus Code $r-s$	Impedance $z_{pq,rs}$
1	1-2(1)	0.6	—	—
4	1-2(2)	0.4	1-2(1)	0.2
2	1-3	0.5	1-2(1)	0.1
3	3-4	0.5	—	—
5	2-4	0.2	—	—

Sol:-

Step (1):-

Start with element 1 which is a branch from $p=1$ to $q=2$

The elements of the bus impedance matrix for the partial network Containing the single branch are

$$Z_{BUS} = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \end{array}$$

Since bus 1 is taken as reference, $Z_{11} = Z_{12} = Z_{21} = 0$ and $Z_{22} = 0.6$

$$\therefore Z_{BUS} = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 0 \\ 0 & 0.6 \end{bmatrix} \end{array} = \begin{array}{c|c} & 2 \\ 2 & \begin{bmatrix} 0.6 \end{bmatrix} \end{array}$$

Step (2):-

Add element '4', which is a 'link', from $p=1$ (Reference) to $q=2$

It is mutually coupled with element '1'.

Now, the augmented impedance matrix with the fictitious node 'l' will be

$$Z_{BUS\,(aug)} = \begin{array}{c|c} & \begin{array}{cc} 2 & l \end{array} \\ \begin{array}{c} 2 \\ l \end{array} & \begin{bmatrix} 0.6 & Z_{2l} \\ Z_{l2} & Z_{ll} \end{bmatrix} \end{array}$$

where,

$$Z_{2l} = Z_{l2} = Z_{12} - Z_{22} + \frac{y_{12(2),12(1)}\,(Z_{12} - Z_{22})}{y_{12(2),12(2)}}$$

$$Z_{ll} = Z_{1l} - Z_{2l} + \left[\frac{1 + y_{12(2),12(1)}\,(Z_{1l} - Z_{2l})}{y_{12(2),12(2)}}\right]$$

$\because Z_{12} = Z_{1l} = 0$, we get

$$Z_{2l} = Z_{l2} = 0 - 0.6 + \left[\frac{y_{12(2),12(1)}\,[0 - 0.6]}{y_{12(2),12(2)}}\right]$$

$$Z_{\ell\ell} = 0 - Z_{2\ell} + \left[\frac{1 + y_{12(2),12(1)} [0 - Z_{2\ell}]}{y_{12(2),12(2)}} \right]$$

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To obtain the values of $y_{12(2),12(1)}$ and $y_{12(2),12(2)}$, the pri[mitive] impedance ~~impedance~~ matrix of the network should be inverted.

$$[Z_{e\sigma,e\sigma}] = \begin{array}{c|cc} & 1\text{-}2(1) & 1\text{-}2(2) \\ \hline 1\text{-}2(1) & 0.6 & 0.2 \\ 1\text{-}2(2) & 0.2 & 0.4 \end{array}$$

$$[y_{e\sigma,e\sigma}] = [Z_{e\sigma,e\sigma}]^{-1} = \begin{array}{c|cc} & 1\text{-}2(1) & 1\text{-}2(2) \\ \hline 1\text{-}2(1) & 2 & -1 \\ 1\text{-}2(2) & -1 & 3 \end{array}$$

$$\therefore \; Z_{2\ell} = Z_{\ell 2} = -0.6 + \frac{(-1)(-0.6)}{3} = -0.6 + 0.2 = -0.4$$

$$Z_{\ell\ell} = +0.4 + \left[\frac{1 + (-1)(0.4)}{3} \right] = 0.4 + 0.2 = 0.6$$

Now, the augmented Z_{BUS} is

$$Z_{BUS(aug)} = \begin{array}{c} \\ 2 \\ \ell \end{array} \begin{array}{c} \begin{array}{cc} 2 & \ell \end{array} \\ \begin{bmatrix} 0.6 & -0.4 \\ -0.4 & 0.6 \end{bmatrix} \end{array}$$

In order to have the effect of added link, it is necessary to eliminate the ℓ^{th} row and column of above augmented Z_{BUS} matrix and recalculate the element Z_{22}. Let new value of Z_{22} be Z'_{22}.

where, $$Z'_{22} = Z_{22(old)} - \frac{Z_{2\ell} Z_{\ell 2}}{Z_{\ell\ell}} = 0.6 - \frac{(-0.4)(-0.4)}{0.6} = 0.3333$$

and thus, the modified Z_{BUS} matrix is

$$Z_{BUS(mod)} = \begin{array}{c} \\ 2 \end{array} \begin{array}{c} 2 \\ \begin{bmatrix} 0.3333 \end{bmatrix} \end{array}$$

Step (3):

Add element '2', which is a branch, from $p=1$ (reference) to $q=3$, mutually coupled with element 1. Now, this adds a new bus and the Z_{BUS} is

$$Z_{BUS} = \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 0.3333 & Z_{23} \\ Z_{32} & Z_{33} \end{bmatrix} \end{array}$$

~~$z_{32} = z_{23} = z_{31} - z_{21}$~~

$$z_{32} = z_{23} = z_{12} + \frac{\begin{bmatrix} y_{13,12(1)} & y_{13,12(2)} \end{bmatrix} \begin{bmatrix} z_{12} - z_{22} \\ z_{12} - z_{22} \end{bmatrix}}{y_{13,13}}$$

$$z_{32} = z_{23} = 0 + \frac{\begin{bmatrix} y_{13,12(1)} & y_{13,12(2)} \end{bmatrix} \begin{bmatrix} z_{12} - z_{22} \\ z_{12} - z_{22} \end{bmatrix}}{y_{13,13}}$$

$$z_{33} = z_{13} + \left[\frac{1 + \begin{bmatrix} y_{13,12(1)} & y_{13,12(2)} \end{bmatrix} \begin{bmatrix} z_{13} - z_{23} \\ z_{13} - z_{23} \end{bmatrix}}{y_{13,13}} \right]$$

and Since, $z_{12} = z_{13} = 0$ [1 is ref node]. Invert the primitive impedance

$z_{23(primitive)} = 0$

matrix to obtain the primitive admittance matrix,

$$[z_{e\sigma,e\sigma}] = \begin{array}{c|ccc} & 1-2(1) & 1-2(2) & 1-3 \\ \hline 1-2(1) & 0.6 & 0.2 & 0.1 \\ 1-2(2) & 0.2 & 0.4 & 0 \\ 1-3 & 0.1 & 0 & 0.5 \end{array}$$ ← primitive impedance matrix

$$[z_{e\sigma,e\sigma}]^{-1} = [y_{e\sigma,e\sigma}] = \begin{array}{c|ccc} & 1-2(1) & 1-2(2) & 1-3 \\ \hline 1-2(1) & 2.0833 & -1.0417 & -0.4167 \\ 1-2(2) & -1.0417 & 3.0208 & 0.2083 \\ 1-3 & -0.4167 & 0.2083 & 2.0833 \end{array}$$

Then.,

$$z_{32} = z_{23} = \frac{\begin{bmatrix} -0.4167 & 0.2083 \end{bmatrix} \begin{bmatrix} 0 - 0.3333 \\ 0 - 0.3333 \end{bmatrix}}{2.0833} = 0.0333$$

$$z_{33} = 0 + \left[\frac{1 + \begin{bmatrix} -0.4167 & 0.2083 \end{bmatrix} \begin{bmatrix} 0 - 0.0333 \\ 0 - 0.0333 \end{bmatrix}}{2.0833} \right] = 0.4833$$

$$\therefore Z_{BUS(mod)} = \begin{array}{c|cc} & 2 & 3 \\ \hline 2 & 0.3333 & 0.0333 \\ 3 & 0.0333 & 0.4833 \end{array}$$

Step(4):-

Add element '3', which is a branch between buses p=3 to ... not Coupled. This adds a new bus. Hence the new elements of ...

$$Z_{24} = Z_{42} = Z_{32} = 0.0333 \quad \left[\because Z_{qi} = Z_{pi} \text{ if no mutual Coupling}\right.$$

$$Z_{34} = Z_{43} = Z_{33} = 0.4833$$

$$Z_{44} = Z_{34} + \underbrace{Z_{34,34}}_{\text{primitive impedance}} = 0.4833 + 0.5 = 0.9833 \left[\because Z_{qq} = Z_{pq} \ldots \text{ if no mutual Coupling}\right]$$

$$\therefore Z_{BUS(modified)} = \begin{array}{c|ccc} & ② & ③ & ④ \\ \hline ② & 0.3333 & 0.0333 & 0.0333 \\ ③ & 0.0333 & 0.4833 & 0.4833 \\ ④ & 0.0333 & 0.4833 & 0.9833 \end{array}$$

Step(5):-

Add element '5', which is a link, from p=2 to q=4, not mutually Coupled. The elements of the l^{th} row and Column of the augmented matrix are,

$$Z_{2l} = Z_{l2} = Z_{22} - Z_{42} = 0.3333 - 0.0333 = 0.3 \quad \left[\because Z_{li} = Z_{pi} - Z_{qi} \text{ for no mutual Coupling}\right]$$

$$Z_{3l} = Z_{l3} = Z_{23} - Z_{43} = 0.0333 - 0.4833 = -0.45$$

$$Z_{4l} = Z_{l4} = Z_{24} - Z_{44} = 0.0333 - 0.9833 = -0.95$$

$$Z_{ll} = Z_{2l} - Z_{4l} + Z_{24,24} = 0.3 + 0.95 + 0.2 = 1.45 \left[\because Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq} \text{ for no mutual Coupling}\right]$$

Now the augmented matrix is

$$Z_{BUS(modified)} = \begin{array}{c|cccc} & ② & ③ & ④ & ⓛ \\ \hline ② & 0.3333 & 0.0333 & 0.0333 & 0.3 \\ ③ & 0.0333 & 0.4833 & 0.4833 & -0.45 \\ ④ & 0.0333 & 0.4833 & 0.9833 & -0.95 \\ ⓛ & 0.3 & -0.45 & -0.95 & 1.45 \end{array}$$

Now we have to eliminate 'l' th row and column elements.

i.e. $Z_{l2}, Z_{l3}, Z_{l4}, Z_{2l}, Z_{3l}, Z_{4l}$ and Z_{ll}. Then modified other elements will be

$$(i=2, j=2)\ Z'_{22} = Z_{22}(old) - \frac{Z_{l2} Z_{2l}}{Z_{ll}} = 0.3333 - \frac{(0.3)(0.3)}{1.45} = 0.2712$$

$$(i=2, j=3)\ Z'_{23} = Z'_{32} = Z_{23}(old) - \frac{Z_{2l} Z_{l3}}{Z_{ll}} = 0.0333 - \frac{(0.3)(-0.45)}{1.45} = 0.1263$$

$$(i=2, j=4)\ Z'_{24} = Z'_{42} = Z_{24}(old) - \frac{Z_{2l} Z_{l4}}{Z_{ll}} = 0.0333 - \frac{(0.3)(-0.95)}{1.45} = 0.2298 \ \cancel{= 0.1885}$$

$$(i=3, j=3)\ Z'_{33} = Z_{33}(old) - \frac{Z_{3l} Z_{l3}}{Z_{ll}} = 0.4833 - \frac{(0.45)(-0.45)}{1.45} = 0.3436$$

$$(i=3, j=4)\ Z'_{34} = Z'_{43} = Z_{34}(old) - \frac{Z_{3l} Z_{l4}}{Z_{ll}} = 0.4833 - \frac{(0.45)(-0.95)}{1.45} = 0.1885$$

$$(i=j=4)\ Z'_{44} = Z_{44}(old) - \frac{Z_{4l} Z_{l4}}{Z_{ll}} = 0.9833 - \frac{(-0.95)(-0.95)}{1.45} = 0.3609$$

Hence, the final modified Z_{BUS} matrix is

$$Z_{BUS}(final) = \begin{array}{c} \\ (2) \\ (3) \\ (4) \end{array} \begin{array}{c} \begin{array}{ccc} (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 0.2712 & 0.1263 & 0.2298 \\ 0.1263 & 0.3436 & 0.1885 \\ 0.2298 & 0.1885 & 0.3609 \end{bmatrix} \end{array}$$

(b) Adding a new element, which is a link, from $p=2$ to $q=4$, mutually coupled with element 5 results in the augmented matrix

$$Z_{BUS} = \begin{array}{c} \\ (2) \\ (3) \\ (4) \\ (l) \end{array} \begin{array}{c} \begin{array}{cccc} (2) & (3) & (4) & (l) \end{array} \\ \begin{bmatrix} 0.2712 & 0.1263 & 0.2298 & Z_{2l} \\ 0.1263 & 0.3436 & 0.1885 & Z_{3l} \\ 0.2298 & 0.1885 & 0.3609 & Z_{4l} \\ Z_{l2} & Z_{l3} & Z_{l4} & Z_{ll} \end{bmatrix} \end{array}$$

where $Z_{li} = Z_{pi} - Z_{qi} + \dfrac{\bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}}$, $i = 2,3,4$

and

$$z_{\ell\ell} = z_{p\ell} - z_{q\ell} + \frac{[1 + y_{pq,\rho\sigma}(z_{p\ell} - \ldots)]}{y_{pq,pq}}$$

The primitive impedance matrix is

$$[z] = \begin{array}{c} \\ 1\text{-}2(1) \\ 1\text{-}2(2) \\ 1\text{-}3 \\ 3\text{-}4 \\ 2\text{-}4(1) \\ 2\text{-}4(2) \end{array} \begin{array}{c} \begin{array}{cccccc} 1\text{-}2(1) & 1\text{-}2(2) & 1\text{-}3 & 3\text{-}4 & 2\text{-}4(1) & 2\text{-}4(2) \end{array} \\ \begin{bmatrix} 0.6 & 0.2 & 0.1 & 0 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0 & 0.1 & 0.3 \end{bmatrix} \end{array}$$

Since the new element is coupled to only one other element, it is sufficient to invert the submatrix for the coupled elements, which is

$$[z_{\rho\sigma,\rho\sigma}] = \begin{array}{c} \\ 2\text{-}4(1) \\ 2\text{-}4(2) \end{array} \begin{array}{c} \begin{array}{cc} 2\text{-}4(1) & 2\text{-}4(2) \end{array} \\ \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix} \end{array}$$

Thus

$$[y_{\rho\sigma,\rho\sigma}] = \begin{array}{c} \\ 2\text{-}4(1) \\ 2\text{-}4(2) \end{array} \begin{array}{c} \begin{array}{cc} 2\text{-}4(1) & 2\text{-}4(2) \end{array} \\ \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \end{array}$$

and

$$z_{2\ell} = z_{\ell 2} = 0.2712 - 0.2298 + \frac{(-2.0)(0.2712 - 0.2298)}{4.0} = 0.0207$$

$$z_{3\ell} = z_{\ell 3} = 0.1263 - 0.1885 + \frac{(-2.0)(0.1263 - 0.1885)}{4.0} = -0.0311$$

$$z_{4\ell} = z_{\ell 4} = 0.2298 - 0.3609 + \frac{(-2.0)(0.2298 - 0.3609)}{4.0} = -0.0656$$

$$z_{\ell\ell} = 0.0207 + 0.0656 + \frac{[1 - 2.0(0.0207 + 0.0656)]}{4.0} = 0.2931$$

The augmented matrix is

$$z_{Bus(aug)} = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ \ell \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & \ell \end{array} \\ \begin{bmatrix} 0.2712 & 0.1263 & 0.2298 & 0.0207 \\ 0.1263 & 0.3436 & 0.1885 & -0.0311 \\ 0.2298 & 0.1885 & 0.3609 & -0.0656 \\ 0.0207 & -0.0311 & -0.0656 & 0.2931 \end{bmatrix} \end{array}$$

Eliminating the 1th row and column,

$$Z'_{22} = 0.2712 - \frac{(0.0207)(0.0207)}{0.2931} = 0.2697$$

$$Z'_{23} = Z'_{32} = 0.1263 - \frac{(0.0207)(-0.0311)}{0.2931} = 0.1285$$

$$Z'_{24} = Z'_{42} = 0.2298 - \frac{(0.0207)(-0.0656)}{0.2931} = 0.2344$$

$$Z'_{33} = 0.3436 - \frac{(-0.0311)(-0.0311)}{0.2931} = 0.3403$$

$$Z'_{34} = Z'_{43} = 0.1885 - \frac{(-0.0311)(-0.0656)}{0.2931} = 0.1816$$

$$Z'_{44} = 0.3609 - \frac{(-0.0656)(-0.0656)}{0.2931} = 0.3462$$

Finally,

$$Z_{BUS} = \begin{array}{c} \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.2697 & 0.1285 & 0.2344 \\ 0.1285 & 0.3403 & 0.1816 \\ 0.2344 & 0.1816 & 0.3462 \end{bmatrix} \end{array}$$

Modification of the Bus Impedance matrix for changes in the network

The bus impedance matrix can be modified to reflect changes in the network. These changes may be addition of elements, removal of elements or changes in the impedances of elements.

If new elements are added to a partial network, new Z_{BUS} is considered as Z'_{BUS}.

The procedure to remove elements or to change the impedances of elements is the same. If the element is removed which is not mutually coupled to any other element, the modified Z_{BUS} can be obtained by adding, in parallel with the element, a link whose impedance is equal to the negative of the impedance of the element to be removed.

If the impedance of an uncoupled element is changed, the modified Z_{BUS} can be obtained by adding a link in parallel with the element such that the equivalent impedance of the two element is the desired value.

When mutually coupled elements are removed or their impedances changed, the Z_{BUS}(modified) can't be obtained by adding a link & using the procedure as used earlier.

However, an equation can be derived for modifying the elements of Z_{BUS} by introducing appropriate changes in the bus currents of the original network to simulate the removal of elements or changes in their impedances.

The performance equation in terms of the new bus currents is

$$\bar{E}'_{BUS} = Z_{BUS}\left(\bar{I}_{BUS} + \Delta\bar{I}_{BUS}\right) \quad \text{—} ①$$

where $\Delta \bar{I}_{BUS}$ is a vector of bus current changes so that $\bar{E}'_{BUS}$ will reflect the desired changes in the network.

An element z'_{ij} of the $Z_{BUS(modified)}$ can be obtained by calculating for the modified n/w the voltage at bus i with a current injected at bus j.

This is equivalent to calculating for the original n/w the voltage at bus i with the same value of current injected at bus j and appropriate changes in currents at the buses which are terminals of the elements being changed.

If the elements μ-ν coupled to elements ρ-σ are removed or their impedances are changed, the corresponding changes in the bus currents are

$$\left.\begin{aligned}
\Delta I_k &= \Delta i_{\mu\nu} & k &= \mu \\
\Delta I_k &= -\Delta i_{\mu\nu} & k &= \nu \\
\Delta I_k &= \Delta i_{\rho\sigma} & k &= \rho \\
\Delta I_k &= -\Delta i_{\rho\sigma} & k &= \sigma \\
\Delta I_k &= 0 & &\forall \text{ other } k
\end{aligned}\right\} \quad \text{—— (2)}$$

Letting the injected current at the jth bus equal to one per unit,

$$\left.\begin{aligned}
I_j &= 1 \\
I_k &= 0 \quad k = 1, 2, \ldots n \\
& \qquad\; k \neq j
\end{aligned}\right. \quad \text{— (3)}$$

From the performance eqn (1),

$$E'_i = \sum_{k=1}^{n} z_{ik}(I_k + \Delta I_k) \quad i = 1, 2, \ldots n$$

Substituting for ΔI_k and I_k from eqns (2) & (3),

$$E'_i = z_{ij} + z_{i\mu}\Delta i_{\mu\nu} - z_{i\nu}\Delta i_{\mu\nu} + \bar{z}_{i\rho}\Delta i_{\rho\sigma} - \bar{z}_{i\sigma}\Delta i_{\rho\sigma}$$

$$\bar{E}'_i = z_{ij} + (\bar{z}_{i\mu} - \bar{z}_{i\nu})\Delta \bar{i}_{\mu\nu} + (\bar{z}_{i\rho} - \bar{z}_{i\sigma})\overline{\Delta i}_{\rho\sigma}$$

Using the Subscript $\alpha\beta$ for network elements μ-ν and ρ-σ,

$$\bar{E}_i' = Z_{ij} + (\bar{Z}_{i\alpha} - \bar{Z}_{i\beta})\,\Delta \bar{i}_{\alpha\beta} \quad i = 1, 2, \cdots n \quad \text{—} \quad (4)$$

From the performance equation of the primitive network,

$$\Delta \bar{i}_{\alpha\beta} = ([y_\delta] - [y_\delta'])\,\bar{v}_{\gamma\delta}' \quad \text{———} \quad (5)$$

where $[y_\delta]$ and $[y_\delta']$ are respectively the square submatrices of the original and modified primitive admittance matrices.

The rows & columns of the submatrices correspond to the network elements μ-ν and ρ-σ. The subscripts of the elements of $(|y_\delta| - |y_\delta'|)$ are $\alpha\beta, \gamma\delta$. The voltage vector in eqn (5) is

$$\bar{v}_{\gamma\delta}' = \bar{E}_\gamma' - \bar{E}_\delta'$$

Substituting for $\bar{E}_\gamma'$ and $\bar{E}_\delta'$ from eqn (4),

$$\bar{v}_{\gamma\delta}' = \bar{Z}_{\gamma j} - \bar{Z}_{\delta j} + ([\bar{Z}_{\gamma\alpha}] - [Z_{\delta\alpha}] - [Z_{\gamma\beta}] + [Z_{\delta\beta}])\,\Delta \bar{i}_{\alpha\beta} \quad \text{—} \quad (6)$$

Substituting from eqn (6) for $\bar{v}_{\gamma\delta}'$ into (5),

$$\Delta \bar{i}_{\alpha\beta} = ([y_\delta] - [y_\delta'])\{\bar{Z}_{\gamma i} - \bar{Z}_{\delta j} + ([Z_{\gamma\alpha}] - [Z_{\delta\alpha}] - [\bar{Z}_{\gamma\beta}] + [Z_{\delta\beta}])\,\Delta \bar{i}_{\alpha\beta}\} \quad \text{——} \quad (7)$$

Solving eqn (7) for $\Delta \bar{i}_{\alpha\beta}$,

$$\Delta \bar{i}_{\alpha\beta} = \{U - ([y_\delta] - [y_\delta'])([Z_{\gamma\alpha}] - [Z_{\delta\alpha}] - [Z_{\gamma\beta}] + [Z_{\delta\beta}])\}^{-1} ([y_\delta] - [y_\delta'])(\bar{Z}_{\gamma j} - \bar{Z}_{\delta j}) \quad \text{———} \quad (8)$$

Designating

$$[\Delta y_\delta] = [y_\delta] - [y_\delta']$$

and

$$[M] = \{U - [\Delta y_\delta]([Z_{\gamma\alpha}] - [Z_{\delta\alpha}] - [Z_{\gamma\beta}] + [Z_{\delta\beta}])\}$$

Eqn (8) becomes

$$\Delta \bar{i}_{\alpha\beta} = [M]^{-1}[\Delta y_\delta](\bar{Z}_{\gamma j} - \bar{Z}_{\delta j}) \quad \text{—} \quad (9)$$

Substituting from eqn (9) for $\bar{\Delta i}_{\alpha\beta}$ into (4),

$$E_i' = Z_{ij} + (\bar{Z}_{i\alpha} - \bar{Z}_{i\beta})[M]^{-1}[\Delta y_\delta](\bar{Z}_{\gamma j} - \bar{Z}_{\delta j})$$

This equation gives, for the original network, the voltage at bus i as a result of injecting one per unit current at bus j and the appropriate current changes at buses μ, ν, ρ and σ to simulate the effect of changes in the elements μ-ν.

Thus from the definition of the Z_{BUS}, the ij^{th} element of the Z_{BUS} (modified) is

$$Z_{ij}' = Z_{ij} + (\bar{Z}_{i\alpha} - \bar{Z}_{i\beta})[M]^{-1}[\Delta y_\delta](\bar{Z}_{\gamma j} - \bar{Z}_{\delta j}) \quad i = 1, 2, \cdots n$$

The process is repeated for each $j = 1, 2, \cdots n$ to obtain all elements of Z_{BUS}'.

(C) The modified elements of this Z_{BUS} matrix for the removal of the network element 2-4(2) mutually coupled to network element 2-4(1) are obtained from

$$Z_{ij}' = Z_{ij} + (\bar{Z}_{i\alpha} - \bar{Z}_{i\beta})[M]^{-1}[\Delta y_\delta](\bar{Z}_{\gamma j} - \bar{Z}_{\delta j}) \quad i, j = 2, 3, 4$$

where μ-ν is 2-4 and ρ-σ is also 2-4 and the indices $\alpha, \gamma = 2, 2$ and $\beta, \delta = 4, 4$.

The original primitive admittance submatrix is

$$[y_\delta] = \begin{array}{c c} & \begin{array}{cc} 2\text{-}4(1) & 2\text{-}4(2) \end{array} \\ \begin{array}{c} 2\text{-}4(1) \\ 2\text{-}4(2) \end{array} & \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \end{array}$$

and the modified primitive admittance matrix is

$$[y_\delta'] = \begin{array}{c c} & \begin{array}{cc} 2\text{-}4(1) & 2\text{-}4(2) \end{array} \\ \begin{array}{c} 2\text{-}4(1) \\ 2\text{-}4(2) \end{array} & \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

Thus

$$[y_\delta] - [y'_\delta] = [\Delta y_\delta] = \begin{array}{cc} & \begin{matrix} 2\text{-}4(1) & 2\text{-}4(2) \end{matrix} \\ \begin{matrix} 2\text{-}4(1) \\ 2\text{-}4(2) \end{matrix} & \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \end{array}$$

Also

$$[M] = \{U - [\Delta y_\delta]([z_{\gamma\alpha}] - [z_{\delta\alpha}] - [z_{\gamma\beta}] + [z_{\delta\beta}])\}$$

where

$$[z_{\gamma\alpha}] = \begin{array}{cc} & \begin{matrix} 2 & 2 \end{matrix} \\ \begin{matrix} 2 \\ 2 \end{matrix} & \begin{bmatrix} 0.2697 & 0.2697 \\ 0.2697 & 0.2697 \end{bmatrix} \end{array}, \quad [z_{\delta\alpha}] = \begin{array}{cc} & \begin{matrix} 2 & 2 \end{matrix} \\ \begin{matrix} 4 \\ 4 \end{matrix} & \begin{bmatrix} 0.2344 & 0.2344 \\ 0.2344 & 0.2344 \end{bmatrix} \end{array}$$

$$[z_{\gamma\beta}] = \begin{array}{cc} & \begin{matrix} 4 & 4 \end{matrix} \\ \begin{matrix} 2 \\ 2 \end{matrix} & \begin{bmatrix} 0.2344 & 0.2344 \\ 0.2344 & 0.2344 \end{bmatrix} \end{array}, \quad [z_{\delta\beta}] = \begin{array}{cc} & \begin{matrix} 4 & 4 \end{matrix} \\ \begin{matrix} 4 \\ 4 \end{matrix} & \begin{bmatrix} 0.3462 & 0.3462 \\ 0.3462 & 0.3462 \end{bmatrix} \end{array}$$

Substituting in the above equation,

$$[M] = \begin{bmatrix} 1.1471 & 0.1471 \\ -0.2942 & 0.7058 \end{bmatrix}, \quad [M]^{-1} = \begin{bmatrix} 0.82753 & -0.17247 \\ 0.34494 & 1.34494 \end{bmatrix}$$

and

$$[M]^{-1}[\Delta y_\delta] = \begin{bmatrix} 1.17247 & -2.34494 \\ -2.34494 & 4.68988 \end{bmatrix}$$

For $i=2$ and $j=2$,

$$z'_{22} = z_{22} + ([z_{22} \;\; z_{22}] - [z_{24} \;\; z_{24}])[M]^{-1}[\Delta y_\delta]\left[\begin{bmatrix} z_{22} \\ z_{22} \end{bmatrix} - \begin{bmatrix} z_{42} \\ z_{42} \end{bmatrix}\right]$$

$$z'_{22} = 0.2697 + [0.0353 \;\; 0.0353] \begin{bmatrix} 1.17247 & -2.34494 \\ -2.34494 & 4.68988 \end{bmatrix} \begin{bmatrix} 0.0353 \\ 0.0353 \end{bmatrix}$$

$$z'_{22} = 0.2697 + 0.0015 = 0.2712$$

For $i=2$ and $j=3$

$$z'_{23} = z_{23} + ([z_{22} \;\; z_{22}] - [z_{24} \;\; z_{24}])[M]^{-1}[\Delta y_\delta]\left[\begin{pmatrix} z_{23} \\ z_{23} \end{pmatrix} - \begin{pmatrix} z_{43} \\ z_{43} \end{pmatrix}\right]$$

$$z'_{23} = 0.1285 + \begin{bmatrix} 0.0353 & 0.0353 \end{bmatrix} \begin{bmatrix} 1.17247 & -2.34494 \\ -2.34494 & 4.68988 \end{bmatrix} \begin{bmatrix} -0.0531 \\ -0.0531 \end{bmatrix}$$

$$z'_{23} = 0.1285 - 0.0022 = 0.1263$$

For $i = 2$ and $j = 4$,

$$z'_{24} = z_{24} + \left(\begin{bmatrix} z_{22} & z_{22} \end{bmatrix} - \begin{bmatrix} z_{24} & z_{24} \end{bmatrix} \right) [M]^{-1} [\Delta y_\delta] \left(\begin{bmatrix} z_{24} \\ z_{24} \end{bmatrix} - \begin{bmatrix} z_{44} \\ z_{44} \end{bmatrix} \right)$$

$$z'_{24} = 0.2344 + \begin{bmatrix} 0.0353 & 0.0353 \end{bmatrix} \begin{bmatrix} 1.17247 & -2.34494 \\ -2.34494 & 4.68988 \end{bmatrix} \begin{bmatrix} -0.1118 \\ -0.1118 \end{bmatrix}$$

$$z'_{24} = 0.2344 - 0.0046 = 0.2298$$

For $i = 3$ and $j = 3$,

$$z'_{33} = z_{33} + \left(\begin{bmatrix} z_{32} & z_{32} \end{bmatrix} - \begin{bmatrix} z_{34} & z_{34} \end{bmatrix} \right) [M]^{-1} [\Delta y_\delta] \left(\begin{bmatrix} z_{23} \\ z_{23} \end{bmatrix} - \begin{bmatrix} z_{43} \\ z_{43} \end{bmatrix} \right)$$

$$z'_{33} = 0.3403 + \begin{bmatrix} -0.0531 & -0.0531 \end{bmatrix} \begin{bmatrix} 1.17247 & -2.34494 \\ -2.34494 & 4.68988 \end{bmatrix} \begin{bmatrix} -0.0531 \\ -0.0531 \end{bmatrix}$$

$$z'_{33} = 0.3403 + 0.0033 = 0.3436$$

For $i = 3$ and $j = 4$,

$$z'_{34} = z_{34} + \left(\begin{bmatrix} z_{32} & z_{32} \end{bmatrix} - \begin{bmatrix} z_{34} & z_{34} \end{bmatrix} \right) [M]^{-1} (\Delta y_\delta) \left(\begin{bmatrix} z_{24} \\ z_{24} \end{bmatrix} - \begin{bmatrix} z_{44} \\ z_{44} \end{bmatrix} \right)$$

$$z'_{34} = 0.1816 + \begin{bmatrix} -0.0531 & -0.0531 \end{bmatrix} \begin{bmatrix} 1.17247 & -2.34494 \\ -2.34494 & 4.68988 \end{bmatrix} \begin{bmatrix} -0.1118 \\ -0.1118 \end{bmatrix}$$

$$z'_{34} = 0.1816 + 0.0069 = 0.1885$$

Power flow study (or) load flow Analysis.

Power flow study is the steady state analysis of the Power system network.

The main objective of Power flow study is:

* To find the unknown bus voltages & its Phase angles in each branch.
* To find the active & reactive Power at each line
* To reduce the losses & control active, reactive Power.
* To increases the efficiency.
* To improve Power factor & voltage regulation.

Bus:

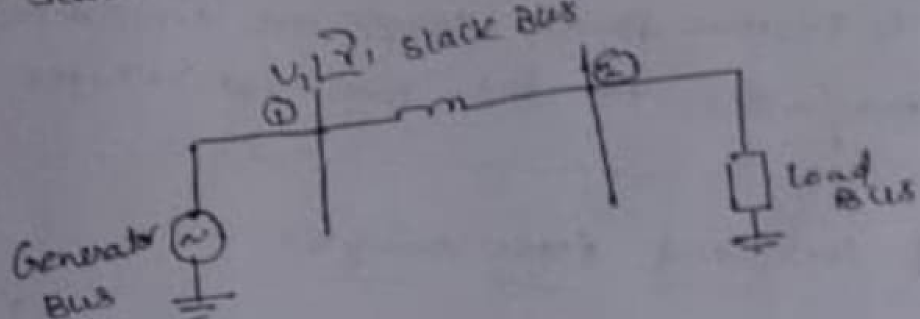
Bus is the vertical line at which several components are connected (i.e. Generators, feeders etc.)

slack bus:

[note: Important parameters are P, Q, |V|, δ i.e. (active power, reactive power, voltage & phase angle]

Buses are classified into three types based on parameters

→ Generator Bus (PV Bus)
→ Load Bus (PQ Bus)
→ Slack Bus (Swing Bus, Reference Bus)



Generator Bus: (PV Bus)

→ Generator Bus is also known as voltage control Bus.

known Parameters: P, V

unknown Parameters: Q, δ

→ Generator Bus is connected at generation side.

Load Bus: (PQ Bus)

→ load Bus is connected at Load side.

known parameters: P, Q

unknown parameters: |V|, δ

Slack Bus: (Reference Bus)

→ Slack Bus is also known as swing Bus

known Parameters: V, δ

unknown Parameters: P, Q

→ slack Bus is the reference bus.

Importance of slack bus:

* Slack bus is the king of all the buses.
* If slack bus is not there we can't find the solutions of active & reactive power.
* Active & Reactive power mainly depends on voltage & phase angle.

[Note: slack bus - Known parameters are v, δ]

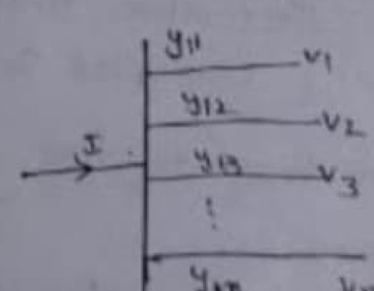
Power flow Equations:

Let us consider the complex power

i.e. $S_i = P_i + jQ_i = \bar{V}_i \bar{I}^*$

$\overset{*}{S}_i = P_i - jQ_i = \overset{*}{V}_i \bar{I}$

[Complex power: The algebric sum of active & reactive power]



$\overset{*}{S}_i = P_i - jQ_i = \overset{*}{V}_i \bar{I}$

By substituting $I = \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k$ we get

$\overset{*}{S}_i = P_i - jQ_i = \overset{*}{V}_i \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k$

By applying KCL

$I = y_{11}v_1 + y_{12}v_2 + \cdots y_{1n}v_n$

$$\boxed{\bar{I} = \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k}$$

$\overset{*}{S}_i = P_i - jQ_i = \overset{*}{V}_i \sum_{i=1}^{n} \bar{y}_{ik} \bar{V}_k$

$= \overset{*}{V}_i \angle \delta_i \sum_{k=1}^{n} \bar{y}_{ik} \angle \theta_{ik} \bar{V}_k \angle \delta_k$

$= \bar{V}_i \angle -\delta_i \sum_{k=1}^{n} \bar{y}_{ik} \angle \theta_{ik} \bar{V}_k \angle \delta_k$

$\overset{*}{S}_i = P_i - jQ_i = \bar{V}_i \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k \angle \theta_{ik} + \delta_k - \delta_i$

$$\boxed{\begin{aligned} \bar{V}_i &= \bar{V}_i \angle \delta_i \\ \bar{V}_k &= \bar{V}_k \angle \delta_k \\ \bar{y}_{ik} &= \bar{y}_{ik} \angle \theta_{ik} \end{aligned}}$$

Load flow solution by using Gauss Seidal Method: (3)

Let us consider

$$S_i^* = P_i - jQ_i = \bar{V}_i \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k \angle \theta_{ik} + \delta_k - \delta_i$$

Seperating real & (reactive power) Imaginary part

$$\boxed{\begin{aligned} P_i &= \bar{V}_i \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k \cos \angle \theta_{ik} + \delta_k - \delta_i \\ Q_i &= -\bar{V}_i \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k \sin \angle \theta_{ik} + \delta_k - \delta_i \end{aligned}}$$ Power flow equations

P_i & Q_i are the power flow equations.

Load flow studies with & without PV buses by using Gauss Seidal Method: → Gauss-seidel method is an iterative process which starts by assigning estimated values to the unknown bus voltages.

→ To reduce the complexity we have some iterative methods

[Iterative methods are Gauss seidal, Newton Rapshon, Decoupled & fast decoupled methods]

→ we have to find solutions easily by using iterative methods.

→ By using Iterative method we have to find new value for each bus at the end of each iteration.

Acceleration factor:

→ Acceleration factor is the value which is used to speed up the convergence of the system.

→ Acceleration factor is used to reduce the iterative techniques. (α lies between 1.6 to 1.8)

Load flow solution by using Gauss Seidal Method:

Let us consider

n variables: $x_1, x_2, x_3 \ldots x_n$

$$x_i = f_i(x_1, x_2, x_3 \ldots x_n)$$

where $i = 1, 2, 3 \ldots n$

for iteration

$$x_i^1 = f_i(x_1^1, x_2^1, \ldots x_n^1)$$

$$x_i^2 = f_i(x_1^2, x_2^2, \ldots x_n^1)$$

$$\vdots$$

$$x_i^r = f_i(x_1^{r-1}, x_2^{r-1} \ldots x_n^{r-1})$$

for 'r'th iteration

$$\Delta x_i^r = x_i^r - x_i^{r-1} \quad (\text{Present iteration} - \text{previous iteration})$$

By using Gauss seidal method for load flow: (without PV buses)

we have to consider complex Power

$$\bar{S}_i = P_i + jQ_i = \bar{V}_i^*$$

$$\bar{S}_i^* = P_i - jQ_i = \bar{V}_i^* \left[\sum_{k=1}^{n} \bar{y}_{ik} V_k = i\right]$$

$$\bar{S}_i^* = P_i - jQ_i = \bar{V}_i^* \sum_{k=1}^{n} \bar{y}_{ik} \bar{V}_k$$

$$\frac{P_i - jQ_i}{\bar{V}_i^*} = \sum_{\substack{k=1 \\ k \neq i}}^{n} \bar{y}_{ik} \bar{V}_k + \bar{y}_{ii} \bar{V}_i$$

$$\bar{y}_{ii} \bar{V}_i = \frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} \bar{y}_{ik} \bar{V}_k$$

$$\bar{V}_i = \frac{1}{\bar{Y}_{ii}}\left[\frac{P_i - jQ_i}{\overset{*}{V}_i} - \sum_{\substack{k=1\\k\neq i}}^{n} \bar{Y}_{ik}\bar{V}_k\right]$$

for r^{th} iteration

$$\bar{V}_i^{r} = \frac{1}{\bar{Y}_{ii}}\left[\frac{P_i - jQ_i}{\overset{*}{V}_i^{\,r-1}} - \sum_{\substack{k=1\\k\neq i}}^{n} \bar{Y}_{ik}\bar{V}_k^{r-1}\right]$$

(or)

$$\bar{V}_i^{r} = \frac{1}{\bar{Y}_{ii}}\left[\frac{P_i - jQ_i}{\overset{*}{V}_i^{\,r-1}} - \sum_{k=1}^{i-1} \bar{Y}_{ik}\bar{V}_k^{r} - \sum_{k=i+1}^{n} \bar{Y}_{ik}\bar{V}_k^{r-1}\right]$$

With PV buses

Let us consider complex power

$$\overset{*}{S}_i = P_i - jQ_i = \overset{*}{V}_i\, i$$

$$P_i - jQ_i = \overset{*}{V}_i \sum_{k=1}^{n} \bar{Y}_{ik}\bar{V}_k$$

When PV buses are present we have to imaginary part

$$Q_i = -\mathrm{Img}\left\{\overset{*}{V}_i \sum_{k=1}^{n} \bar{Y}_{ik}\bar{V}_k\right\}$$

$$Q_i = -\mathrm{Img}\left\{\bar{V}_i \angle -\delta_i \sum_{k=1}^{n} \bar{Y}_{ik}V_k^{r-1}\right\}$$

for 'r^{th}' iteration

$$Q_i^{r} = -\mathrm{Img}\left\{\bar{V}_i \angle -\delta_i^{\,r-1} \sum_{\substack{k=1\\k\neq i}}^{n} \bar{Y}_{ik}\bar{V}_k^{r-1}\right\}$$

(or)

$$Q_i^{r} = -\mathrm{Img}\left\{\bar{V}_i \angle -\delta_i^{\,r-1}\left[\sum_{k=1}^{i-1} \bar{Y}_{ik}\bar{V}_k^{r} + \sum_{k=i+1}^{n} \bar{Y}_{ik}\bar{V}_k^{r-1}\right]\right\}$$

$$Q_i^{r} = -\mathrm{Img}\left\{ \bar{V}_i \, \underline{\left|-\delta_i^{r-1}\right.} \left[\sum_{k=1}^{i-1} \bar{y}_{ik} \bar{V}_k^{r} + \sum_{k=i+1}^{n} \bar{y}_{ik} \bar{V}_k^{r-1} \right] \right\}$$

$$Q_{i\,min} \le Q_i^{r} \le Q_{i\,max}$$

$$Q_{i\,min} < Q_i^{r} \quad \text{i.e. } Q_{i\,min} = Q_i^{r}$$

$$Q_{i\,max} > Q_i^{r} \quad \text{i.e. } Q_{i\,max} = Q_i^{r}$$

Line losses:

For finding line losses we have to consider the complex power

i.e. $S_i = P_i + jQ_i = V_i I^*$

$$P_i + jQ_i = V_{Bus}^{T} I_{Bus}^{*} \rightarrow (1)$$

we know that $V = Z I$

$$\boxed{V_{Bus} = Z_{Bus} I_{Bus}}$$

Substitute 'V_{Bus}' in eq(1)

$$P_i + jQ_i = (Z_{Bus} I_{Bus})^{T} I_{Bus}^{*}$$

$$P_i + jQ_i = Z_{Bus}^{T} I_{Bus}^{T} I_{Bus}^{*}$$

$$P_i + jQ_i = Z_{Bus}^{T} I_{Bus}^{T} (I_p - jI_q) \quad \begin{bmatrix} Z = R + jX \\ I = I_p + jI_q \end{bmatrix}$$

$$= Z_{Bus} I_{Bus}^{T} (I_p - jI_q) \quad [Z_{Bus}^{T} = Z_{Bus}]$$

$$P_i + jQ_i = (R + jX)(I_p + jI_q)^{T} (I_p - jI_q)$$

Algorithm:

Algorithm is step by step procedure.

Algorithm for Gauss Seidel method:

step-1:

Read the given data.

(i.e PV Bus, PQ Bus)

step-2:

Y-Bus formation

step-3:

Make Initial assumptions

i.e. V_i^0 for $i = (m+1), (m+2) \ldots$

where $i = 1, 2, 3, 4 \ldots n$

step-4:

set iteration count $r = 1$ $|V_{max}| = 0$

step-5:

set bus count $i = 2$

step-6:

If bus type is PQ go to step-11 else go to next step.

step-7:

If bus type is PV. Calculate Q_i^{r+1} & check whether Q_i^{r+1} is within limits or not

step-8:

If Q_i^{r+1} is within limit then go to step 8 else go to next step.

Step-9:

If $Q_i^{r+1} > Q_{imax}$ then assign Q_{imax} to Q_i^{r+1} & calculate V_i^{r+1}

Step-10:

else if $Q_i^{r+1} < Q_{imin}$ then assign Q_{imin} to Q_i^{r+1} & calculate V_i^{r+1}

Step-11:

calculate V_i^{r+1} & update with new voltage value, assign V_{inew} to V_i

Step-12:

Set advance bus count $i=i+1$

Step-13:

If $i \leq n$ then go to step 6

Step-14:

Iteration count $r=r+1$

Step-15:

If $\Delta V_{max} \geq \epsilon$ & $r \leq i$ then go to step 5.

Step-16:

Evaluate the line flow, slack bus & paint the line voltages & line flows (or) losses.

$$S_i = V_i \sum_{i=1}^{n} \bar{Y}_{ik} \bar{V}_k$$

Advantages of Gauss Seidal Method:

→ simple.
→ computer memory requirement less
→ computation time is less.

Disadvantages of Gauss Seidal method:

→ Rate of convergence is low.
→ Number of iterations is low.
→ choice of slack bus effects the convergence.

UNIT - IV

Unsymmetrical Fault Analysis

Various types of unsymmetrical faults that occur in power systems are;

Shunt Type faults

(i) Single line-to-ground (LG) fault

(ii) Line-to-Line fault (LL)

(iii) Double Line-to-Ground (LLG) fault

Series Type Faults

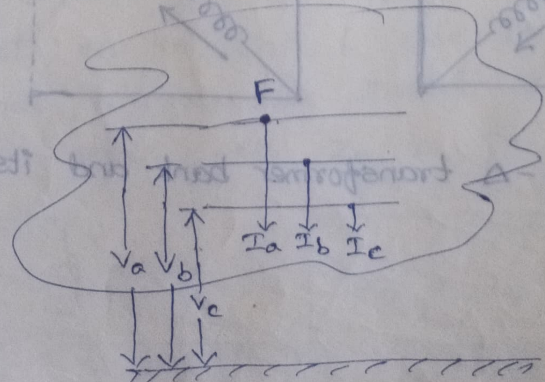
(i) Open Conductor (one or two conductors open) fault

The unsymmetrical fault analysis is important for relay setting, single-phase switching and system stability studies.

Symmetrical Component method is a powerful tool for study of unsymmetrical fault.

Symmetrical Component Analysis of Unsymmetrical faults

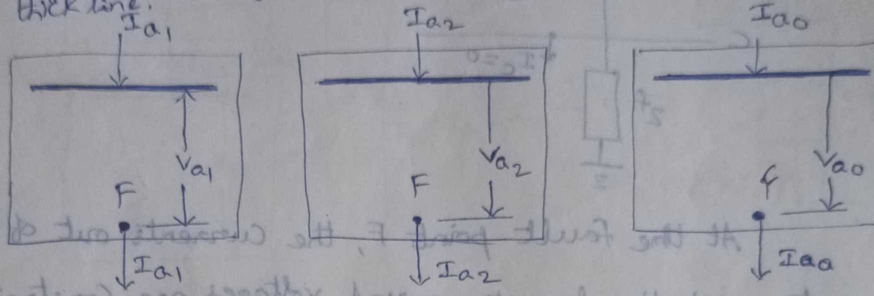
Consider a general power network as shown below. It is assumed that a shunt type fault occurs at a point 'F' in the system, as a result of which currents I_a, I_b and I_c flow out of the system, and V_a, V_b and V_c are voltages of lines a, b, c w.r.t. ground.



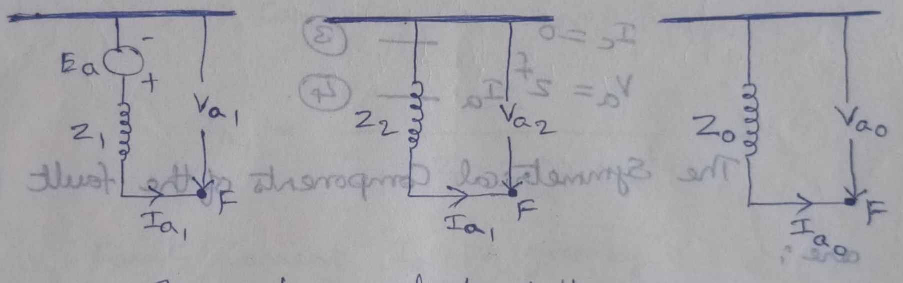
Let us also assume that, the system is operating at no load before the occurence of a fault.

Therefore, positive sequence voltage of all synchronous machines will be identical and will equal the prefault voltage at F. Let this voltage is labelled as E_a'

As seen from F, the power system will present positive, negative and zero sequence networks, which are schematically represented by following figures. The ref. bus is represented by a thick line.



Sequence networks as seen from the fault point F.



Thevenin's equivalents of the sequence networks as seen from fault point F

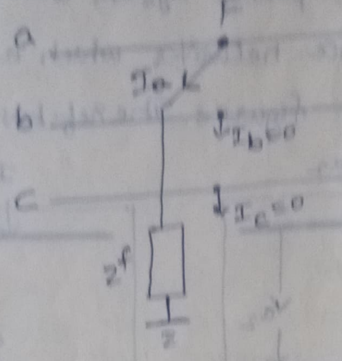
The sequence voltages at 'F' can be expressed in terms of sequence currents and Thevenin's sequence impedances as,

$$\begin{bmatrix} V_{a_1} \\ V_{a_2} \\ V_{a_0} \end{bmatrix} = \begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \begin{bmatrix} I_{a_1} \\ I_{a_2} \\ I_{a_0} \end{bmatrix} \quad — (1)$$

Depending upon the type of fault, the sequence currents and voltages are constrained, leading to a particular connection of sequence networks.

Single Line-to-Ground (L-G) fault

Following figure shows a line-to-ground fault at 'F' in a power system through a fault impedance Z^f. Let us consider that the fault occurs on phase 'a'.



At the fault point F, the currents out of the power system and the line-to-ground voltages are constrained as follows:

$$I_b = 0 \quad \text{——— (2)}$$

$$I_c = 0 \quad \text{——— (3)}$$

$$V_a = Z^f I_a \quad \text{——— (4)}$$

The symmetrical components of the fault currents are:

$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

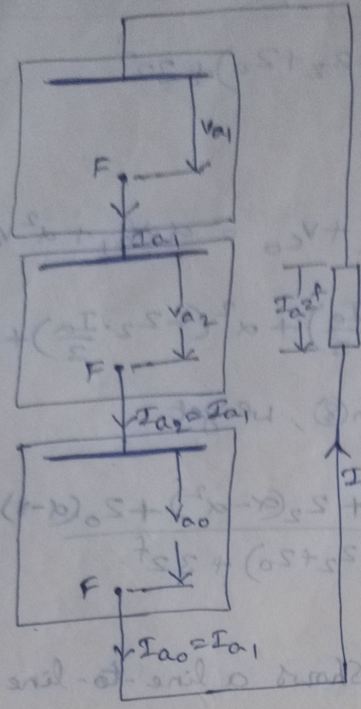
Now, $I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} \cdot I_a$ ——— (5)

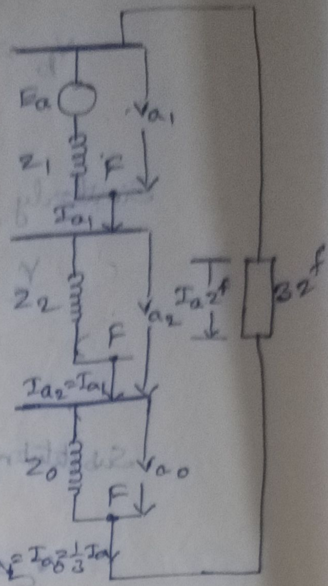
Expressing eqn (4) in terms of symmetrical components, we have

$$V_{a1} + V_{a2} + V_{a0} = Z^f I_a = 3 Z^f \cdot I_{a1} \quad \text{— (6)}$$

From eqns (5) & (6), all sequence currents are equal and the sum of sequence voltages equals $3 Z^f \cdot I_{a1}$.

Now, the sequence networks can be drawn as shown below:





In terms of the Thevenin's equivalent of sequence networks, we can write from above figures as

$$I_{a1} = \frac{E_a}{(Z_1 + Z_2 + Z_0) + 3Z^f} \quad \text{--- (7)}$$

$\therefore$ Fault Current I_a is given by

$$I_a = \frac{3E_a}{(Z_1 + Z_2 + Z_0) + 3Z^f} \quad \text{--- (8)}$$

Substituting V_{a1}, V_{a2}, V_{a0} in equation (1),

$$(E_a - I_{a1} Z_1) + (-I_{a2} Z_2) + (-I_{a0} Z_0) = 3Z^f I_{a1}$$

$$\left[(Z_1 + Z_2 + Z_0) + 3Z^f\right] I_{a1} = E_a$$

(or)

$$I_{a1} = \frac{E_a}{(Z_1 + Z_2 + Z_0) + 3Z^f}$$

The voltage of line 'b' to ground under fault condition is

$$V_b = \alpha^2 V_{a1} + \alpha V_{a2} + V_{a0}$$

$$= \alpha^2 \left(E_a - \frac{I_a}{3} \cdot Z_1\right) + \alpha\left(-Z_2 \cdot \frac{I_a}{3}\right) + \left(-Z_0 \cdot \frac{I_a}{3}\right)$$

Substituting for I_a from eqn ⑧, we get

$$V_b = E_a \cdot \frac{3\alpha^2 \cdot Z^f + Z_2(\alpha^2-\alpha) + Z_0(\alpha^2-1)}{(Z_1+Z_2+Z_0)+3Z^f}$$

Similarly,

$$V_c = V_{c_1} + V_{c_2} + V_{c_0} = \alpha V_{a_1} + \alpha^2 V_{a_2} + V_{a_0}$$

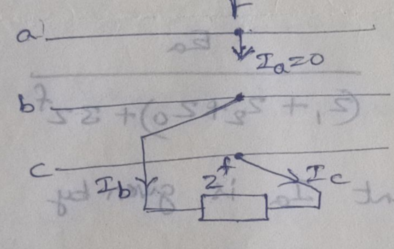
$$= \alpha \cdot \left(E_a - Z_1 \cdot \frac{I_a}{3}\right) + \alpha^2\left(-Z_2 \cdot \frac{I_a}{3}\right) + \left(-Z_0 \cdot \frac{I_a}{3}\right)$$

Substituting for I_a from Eqn ⑧, we get

$$V_c = E_a \cdot \frac{3\alpha Z^f + Z_2(\alpha-\alpha^2) + Z_0(\alpha-1)}{(Z_1+Z_2+Z_0)+3Z^f}$$

Line-to-Line fault

Following figure shows a line-to-line fault at 'F' in a power system on phases b and c through a fault is on phases 'b' and 'c'.



The currents and voltages at the fault can be expressed as

$$I_p = \begin{bmatrix} I_a = 0 \\ I_b \\ I_c = -I_b \end{bmatrix}; \quad V_b - V_c = I_b \cdot Z^f \quad — (10)$$

The Symmetrical Components of the fault currents are:

$$\begin{bmatrix} I_{a_1} \\ I_{a_2} \\ I_{a_0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

from which we get

$$I_{a_2} = -I_{a_1} \quad — (11)$$

$$I_{a_0} = 0 \quad — (12)$$

The Symmetrical Components of voltages at 'F' under fault are:

$$\begin{bmatrix} V_{a_1} \\ V_{a_2} \\ V_{a_0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b - Z^f I_b \end{bmatrix} \quad - (13)$$

Writing the first two equations, we have

$$3V_{a_1} = V_a + (\alpha+\alpha^2)V_b - \alpha^2 Z^f . I_b$$

$$3V_{a_2} = V_a + (\alpha+\alpha^2)V_b - \alpha Z^f . I_b$$

we get

$$3(V_{a_1} - V_{a_2}) = (\alpha - \alpha^2) Z^f I_b = j\sqrt{3}.Z^f.I_b \quad - (14)$$

Now,

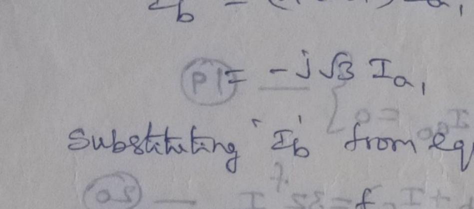
$$I_b = (\alpha^2 - \alpha) I_{a_1} \quad (\because I_{a_2} = -I_{a_1} \ \& \ I_{a_1} = 0)$$

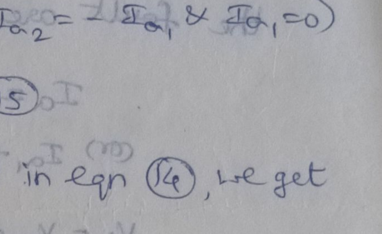
$$= -j\sqrt{3}\, I_{a_1} \quad - (15)$$

Substituting 'I_b' from eqn (15) in eqn (14), we get

$$V_{a_1} - V_{a_2} = Z^f . I_{a_1} \quad - (16)$$

Eqns (11) to (16) suggest parallel connection of positive & Negative sequence networks through a series impedance Z^f as shown in following figures. Since $I_a = 0$, the zero sequence network is unconnected.





In terms of Thevenin's equivalents, we get

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z^f} \quad - (17)$$

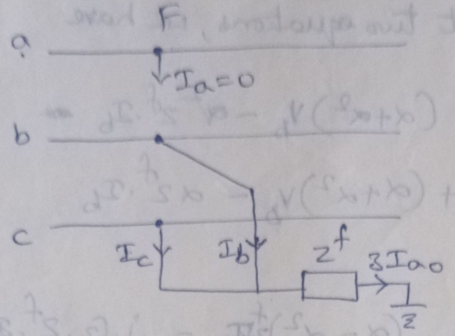
From eqn 15, we get

$$I_b = -I_c = \frac{-j\sqrt{3}\,E_a}{Z_1 + Z_2 + Z^f} \quad - (18)$$

From I_{a_1}, we can calculate V_{a_1} and V_{a_2} from which voltages at the fault can be found.

Double Line-to-Ground Fault (LLG) fault

Following figure shows a double line-to-ground fault at 'F' in a power system. The fault may in general have an impedance, Z^f as shown.



The current and voltage (to ground) conditions at the fault are expressed as

$$I_a = 0$$
$$\text{(or)}\ I_{a1} + I_{a2} + I_{a0} = 0 \quad \text{--- (19)}$$

$$V_b = V_c = Z^f (I_b + I_c) = 3Z^f I_{a0} \quad \text{--- (20)}$$

The symmetrical components of voltages are given by

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{--- (21)}$$

from which it follows that

$$V_{a1} = V_{a2} = \frac{1}{3}\left[V_a + (\alpha+\alpha^2) V_b\right] \quad \text{--- (22 a)}$$

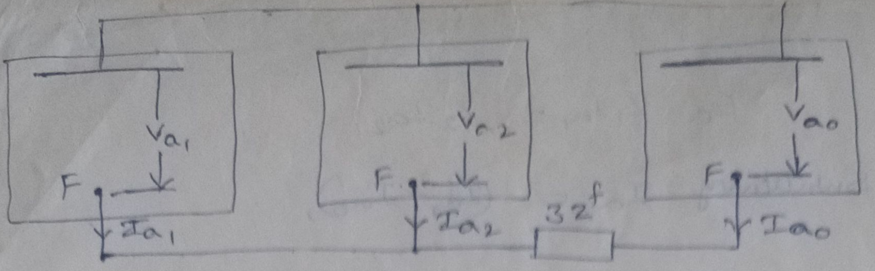
$$V_{a0} = \frac{1}{3}\left[V_a + 2V_b\right] \quad \text{--- (22 b)}$$

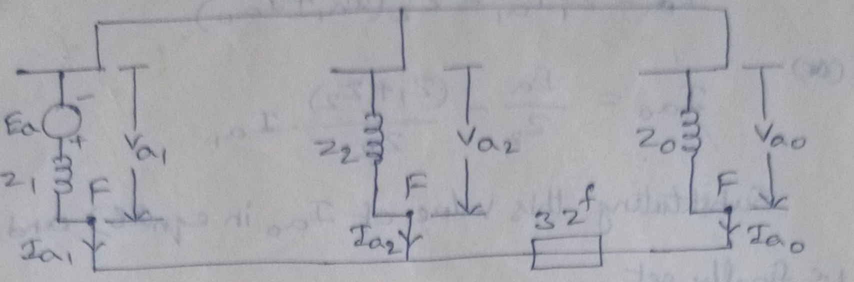
From eqns (22 a) & (22 b),

$$V_{a0} - V_{a1} = \frac{1}{3}(2 - \alpha - \alpha^2) V_b = V_b = 3Z^f I_{a0}$$

$$\text{(or)}\ V_{a0} = V_{a1} + 3Z^f I_{a0} \quad \text{--- (23)}$$

From eqns (19), (22a) & (23), we can draw the connection of sequence networks as shown in following figures.





In terms of the Thevenin's equivalents, we can write

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 \,||\, (Z_0 + 3Z^f)}$$

$$= \frac{E_a}{Z_1 + Z_2(Z_0+3Z^f)/(Z_2+Z_0+3Z^f)} \quad \text{—(24)}$$

Substituting for V_{a1}, V_{a2} and V_{a0} in terms of E_a in eqn ① and premultiplying both sides by Z^{-1} (inverse of sequence impedance matrix), we get

$$\begin{bmatrix} Z^{-1} & 0 & 0 \\ 0 & Z_2^{-1} & 0 \\ 0 & 0 & Z_0^{-1} \end{bmatrix} \begin{bmatrix} E_a - Z_1 I_{a1} \\ E_a - Z_1 I_{a1} \\ E_a - Z_1 I_{a1} + 3Z^f I_{a0} \end{bmatrix}$$

$$= \begin{bmatrix} Z_1^{-1} & 0 & 0 \\ 0 & Z_2^{-1} & 0 \\ 0 & 0 & Z_0^{-1} \end{bmatrix} \begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} \quad \text{— (25)}$$

premultiplying both sides by row matrix $[1 \ 1 \ 1]$ and using eqns (19) and (20), we get

$$-\frac{3Z^f}{Z_0} \cdot I_{a0} + \left[1 + \frac{Z_1}{Z_0} + \frac{Z_1}{Z_2}\right] I_{a1} = \left[\frac{1}{Z_2} + \frac{1}{Z_0}\right] E_a \quad \text{—(26)}$$

From Eqn(22a), we have

$$E_a - Z_1 I_{a_1} = -Z_2 I_{a_2}$$

Substituting $I_{a_2} = -(I_{a_1} + I_{a_0})$,

$$E_a - Z_1 I_{a_1} = Z_2 (I_{a_1} + I_{a_0})$$

(or)

$$I_{a_0} = \frac{E_a}{Z_2} - \frac{(Z_1 + Z_2)}{Z_2} \cdot I_{a_1}$$

Substituting this value of I_{a_0} in eqn(26) and Simplifying, we finally get

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2(Z_0 + 3Z^f) / (Z_2 + Z_0 + 3Z^f)}$$

UNIT-V :- Power System steady state stability Analysis

Introduction

When power system is subjected to some form of disturbance, there is a tendency for the system to develop restoring force to bring it to a normal (or) stable condition.

This ability of a system to reach a normal (or) stable condition after being disturbed is called "stability".

methods impr stabi

Types of stability

Depending upon the magnitude of the disturbance, the stability is divided into three main categories. These are:

(i) Steady state stability
(ii) Transient state stability and
(iii) Dynamic stability.

The 'steady state stability' is the ability of the system to bring it to a stable condition after a small disturbance. It is basically concerned with the effect of gradual variation of load.

The 'transient stability' is the ability of a system to bring it to a stable condition after a large disturbance. It is concerned with sudden ~~and~~ large changes in the network conditions. The large disturbances can occur due to sudden load change, switching operation and faults with subsequent circuit isolation.

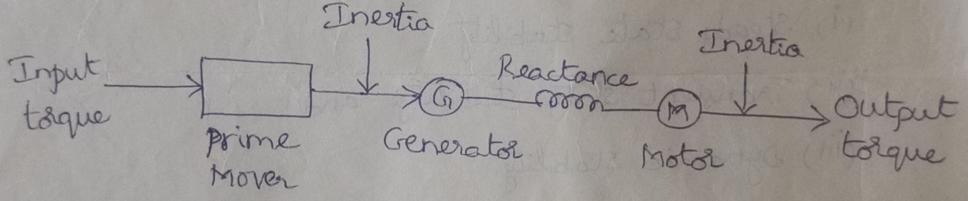
'Dynamic stability' is an extension of the steady state stability. It is concerned with small disturbances lasting for a long time with the inclusion of automatic control devices. This stability can significantly improved through the use of power system stabilizers and study has to Carried out for 5-10 seconds & some times upto 30 sec

Steady state Stability Limit

The "steady state stability limit" is the max power that can be transferred without the system becom unstable when a small disturbance occurs in the powe system network.

Essential factors in the Stability Problem

Following figure shows the one line diagram of a two machine system.



There are two types of factors affecting stability of a power system. They are (i) mechanical and (ii) electrical.

(i) Mechanical factors:-

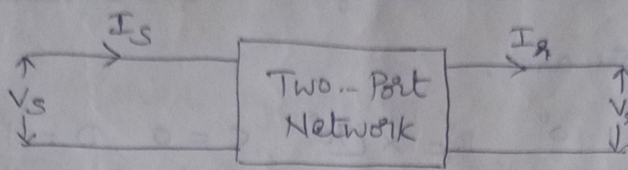
(a) Prime mover input torque
(b) Inertia of prime mover and generator
(c) Inertia of motor and shaft load
(d) shaft load output torque

(ii) Electrical factors:-

(a) Internal voltage of Synchronous generator
(b) Reactance of the system including generator, line and motor.
(c) Internal voltage of Synchronous motor

Expression For steady state Power

The expression for steady state power which can be received (or) transmitted over a line in terms of V_S' and V_r' can be derived as shown below:



The network equations in terms of ABCD parameters are given by

$$V_S = A V_r + B I_r \quad — 1(a)$$

$$I_S = C V_r + D I_r \quad — 1(b)$$

$$\therefore I_r = \frac{V_S}{B} - \frac{A}{B} \cdot V_r \quad — (2)$$

Here A, B, C & D parameters can be defined as,

$$A = |A| \angle \alpha, \quad B = |B| \angle \beta, \quad V_S = V_s \angle \delta, \quad V_r = V_r \angle 0^\circ$$

From eqn (2),

$$I_r = \frac{V_S \angle \delta}{B \angle \beta} - \frac{A \angle \alpha}{B \angle \beta} V_r \angle 0^\circ$$

$$I_r = \left|\frac{V_S}{B}\right| \angle \delta - \beta - \frac{A}{B} \cdot V_r \angle \alpha - \beta \quad — (3)$$

Now, $$I_r^* = \left|\frac{V_S}{B}\right| \angle \beta - \delta - \left|\frac{A}{B} \cdot V_r\right| \angle \beta - \alpha \quad — (4)$$

The complex power at receiving end is given by

$$S_r = P_r + j Q_r = V_r \cdot I_r^* \quad — (5)$$

Substituting V_r, I_r^* values in eqn (5),

$$S_r = P_r + j Q_r = V_r \angle 0^\circ \cdot \left[\left|\frac{V_S}{B}\right| \angle \beta - \delta - \left|\frac{A V_r}{B}\right| \angle \beta - \alpha \right]$$

$$S_r = \frac{V_S V_r}{B} \angle \beta - \delta - \frac{A V_r^2}{B} \cdot \angle \beta - \alpha \quad — (6)$$

Separating real and reactive powers of eqn (6),

Real power, $P_r = \dfrac{V_s \cdot V_r}{B}\cos(\beta-\delta) - \dfrac{A V_r^2}{B}\cos(\beta-\alpha)$ —

and Reactive power, $Q_r = \dfrac{V_s V_r}{B}\sin(\beta-\delta) - \dfrac{A V_r}{B}\sin(\beta-\alpha)$ — (8)

Neglecting the resistance of the line, the values of ABCD parameters are,

$$A = 1\angle 0^\circ, \quad B = X\angle 90^\circ, \quad C = 0, \quad D = 1\angle 0^\circ$$

Substituting the values in eqn (7), the receiving end power becomes,

$$P_r = \frac{V_s V_r}{X}\sin\delta - \frac{V_r^2}{X}\cos 90^\circ$$

$$\boxed{P_r = \frac{V_s V_r}{X}\sin\delta} \quad \text{—— (9)}$$

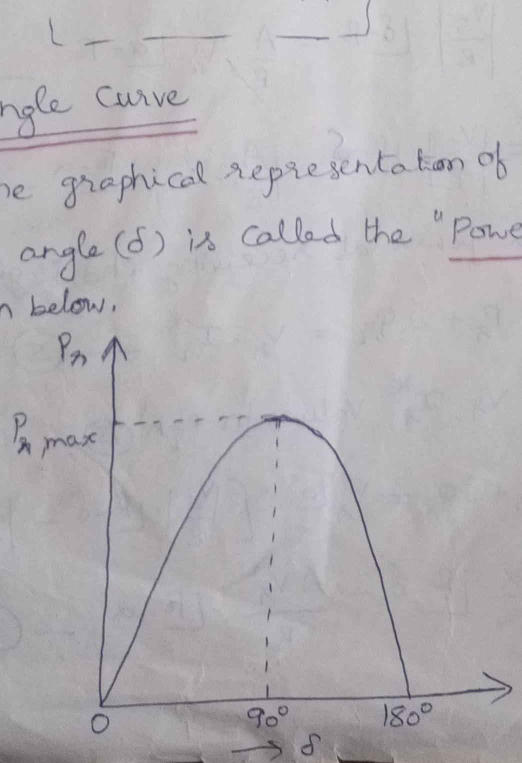
Eqn (9) states that power ~~can be~~ transmitted to receiving end depends upon the reactance of the system and the angles between the two rotors.

If $\delta = 90^\circ$, the power transferred is maximum.

i.e. $\boxed{P_{r,max} = \dfrac{V_s V_r}{X}}$ —— (10)

Power Angle Curve

The graphical representation of power received (P_r) and the load angle (δ) is called the "Power angle curve". It is as shown below.



Neglecting the line resistance and shunt admittance of the transmission line, the power received transmitted is a ~~function of~~ 'sine' function.

(8) The maximum power value will occur at $\delta = 90°$ and its value is $\frac{V_s V_r}{X}$.

Considering that the resistance is present, which is usually the case 'β' will be less than 90° and the maximum power received will occur at $\delta = \beta$.

$$P_r = \frac{V_s V_r}{B}\cos(\beta-\delta) - \frac{AV_r^2}{B}\cos(\beta-\alpha) \quad \text{— (11)}$$

For present case,

$$A = 1\angle 0°,\ B = Z\angle\beta \text{ and } \beta = \tan^{-1}\left(\frac{X}{R}\right)$$

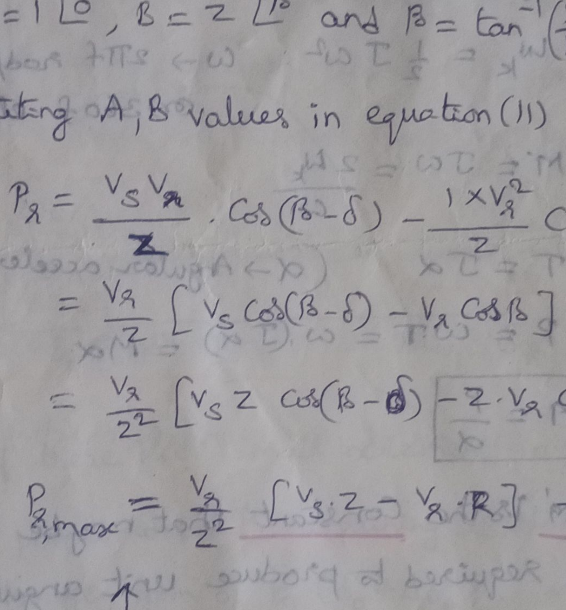
Substituting A, B values in equation (11)

$$P_r = \frac{V_s V_r}{Z}\cdot\cos(\beta-\delta) - \frac{1\times V_r^2}{Z}\cos(\beta)$$

$$= \frac{V_r}{Z}\left[V_s\cos(\beta-\delta) - V_r\cos\beta\right]$$

$$= \frac{V_r}{Z^2}\left[V_s Z\cos(\beta-\delta) \boxed{-Z\cdot V_r\cos\beta}\right] \quad \left[\because \cos\beta = \frac{R}{Z}\right]$$

$$P_{r,max} = \frac{V_r}{Z^2}\left[V_s\cdot Z - V_r\cdot R\right] \quad \text{— (12)}$$



and the maximum power occurs when $\delta = \beta$, the graph will be as shown in above figure.

Constants of the Rotating Machines M and H

The transient conditions of synchronous machine depend mostly on the mechanical constant of the rotor or the prime mover.

Let m = the mass of the rotor in kg.

r = radius of the rotor in m.

W_k = kinetic energy of the rotor in Joules

J = Moment of inertia in kg-m^2

With all other terms ($M \to$ Angular Momentum, $T \to$ Torque) already defined, we have

$$J = m r^2$$

$$W_k = \frac{1}{2} J \omega^2 \qquad \omega \to 2\pi f \text{ rad/sec (or) } 360 f \text{ elec-deg/sec}$$

$$M = J\omega = \frac{2 W_k}{\omega}$$

$$T = J\alpha \qquad (\alpha \to \text{Angular acceleration rad/sec}^2)$$

$$P = \omega T = \omega (J\alpha) = M\alpha$$

$$\therefore \boxed{M = \frac{P}{\alpha}}$$

Thus 'M' is the constant that may be defined as the power in MW required to produce unit angular acceleration.

Another constant 'H', called 'inertia constant', is more frequently used by the manufacturers. It is defined as "the stored kinetic energy at synchronous speed per volt-ampere rating of the machine".

If 'W_k' be the kinetic energy in Mega Joules and 'S' is the rating of the machine in MVA, then

$$H = \frac{W_k}{S} = \frac{\omega . M}{2S} = \frac{2\pi f . M}{2S} = \frac{\pi f M}{S}$$

$$M = \frac{HS}{\pi f} \text{ Mega Joules-sec/elec. radian}$$

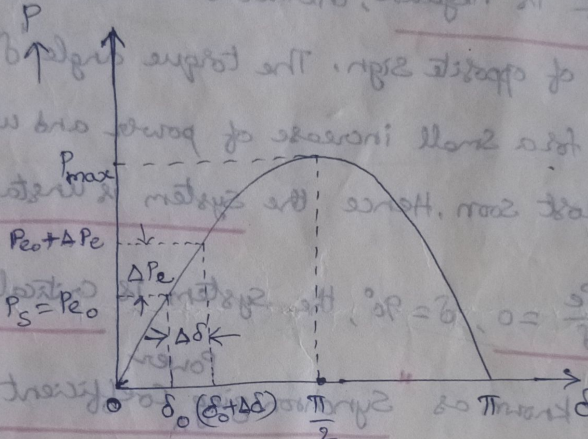
$$M = \frac{HS}{180 f} \text{ Mega Joules-sec/elec. degree}$$

The value of angular momentum 'M' varies over a wide range of MVA for a given type of machine and prime mover but, the value of 'H' is fairly constant.

Steady state Stability

Steady state stability limit is defined as the maximum power that can be transferred through the power system network without loss of synchronism.

Consider a power system operating with steady power transfer, $P_{e_0} = P_s$ with corresponding torque angle δ_0, as shown below:



If there is a small increment in power ΔP_e while the input from prime mover P_s remaining constant, causing the torque angle to change from δ_0 to $(\delta_0 + \Delta\delta_0)$.

Then $\Delta P_e = \frac{\partial P_e}{\partial \delta} \cdot \Delta\delta$

$$M \cdot \frac{d^2 \Delta\delta}{dt^2} = P_s - (P_{e_0} + \Delta P_e) = -\Delta P_e \quad [\because P_s = P_{e_0}]$$

$$\Rightarrow \left[M \frac{d^2}{dt^2} + \frac{\partial P_e}{\partial \delta} \right] \Delta\delta = 0$$

$$\Rightarrow \left[M D^2 + \frac{\partial P_e}{\partial \delta} \right] \Delta\delta = 0$$

where $D = \frac{d}{dt}$

The system stability to small changes is determined from the characteristic equation.

The characteristic equation is

$$MD^2 + \left(\frac{\partial P_e}{\partial \delta}\right) = 0$$

$$\Rightarrow D^2 + \frac{\partial P_e}{M \cdot \partial \delta} = 0$$

whose roots are, $D = \pm \left[\frac{-(\partial P_e/\partial \delta)}{M}\right]^{1/2}$

As long as $\frac{\partial P_e}{\partial \delta}$ is positive, the roots are imaginary and conjugate. The system oscillates about a final value and the system is stable for small incremental change in power.

When $\frac{\partial P_e}{\partial \delta}$ is negative, the roots are real and are equal in magnitude but of opposite sign. The torque angle "δ" increases without control, for a small increase of power and ultimately, synchronism is lost soon. Hence the system is unstable.

When $\frac{\partial P_e}{\partial \delta} = 0$, $\delta = 90°$, the system is critically stable.

Here, '$\frac{\partial P_e}{\partial \delta}$' is known as "Synchronizing Power Coefficient." It is also called as stiffness of synchronous machine.

We know that, $P_e = \frac{EV}{X} \sin \delta$

$\therefore \frac{\partial P_e}{\partial \delta} = \frac{EV}{X} \cos \delta$, assuming E & V are constant.

The system is unstable if $\frac{|E||V|}{X} \cos \delta_0 < 0$ (or) $\delta_0 > 90°$

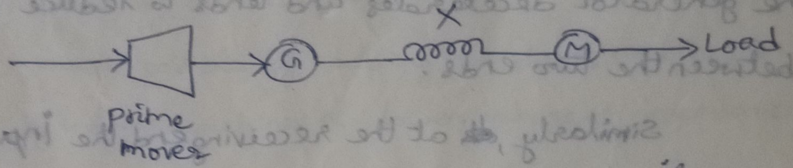
The maximum power that can be transferred without loss of stability occurs at $\delta_0 = 90°$ and is given by

$$P_{max} = \frac{|E||V|}{X}$$

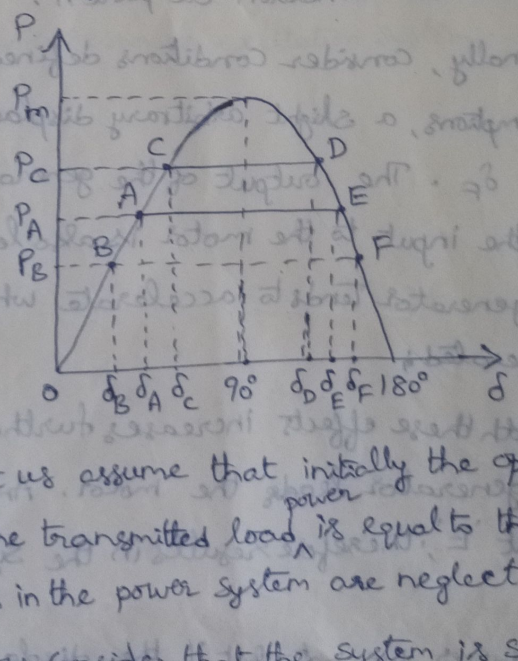
Graphical Approach for determination of steady state Stability

An actual power system would consists of a large number of machines located at different points and as such multi-machine stability problems are too involved and generally calculations are carried out in a digital computer.

Let us consider two machines are connected as shown in following figure. One is acting as synchronous generator feeding to other one acting as synchronous motor through a line reactance.



When the motor is synchronized and is feeding the load, its power input is equal to the output neglecting losses. The power angle curve is as shown in below.



Let us assume that initially the operating point is 'A'. At this point, the transmitted load power is equal to the torque to be overcome, if all losses in the power system are neglected.

Now, consider that the system is subjected to a slight arbitrary displacement in angle from δ_A to δ_B. Then, the power transmitted will change from P_A to P_B.

but the el.

Here, the prime mover input has not changed but the el. output having decreased, the generator rotor will tend to accelerate.

At the receiving end, the motor input is P_B which is ... than shaft output, so that motor tends to decelerate. Th. actions of rotors at both ends tends to increase the angle b. the sending and receiving ends, thereby developing resto. forces and the system returns to the original operating condi. as at point 'A'.

Next consider the system is subjected to an arb. movement which increases the angle from δ_A to δ_C.

In this case, the output being greater than the inp. the generator decelerates and tends to reduce the angle between the two ends.

Similarly, at the receiving end the input to the motor is greater than shaft torque, the motor thus acceler. and tends to decrease the angle between the two ends. Thu. restoring forces are set up, in the system indicate a stable operating point for condition at point 'A'.

Finally, consider conditions defined by point 'E'. As before assumptions, a slight arbitrary displacement in angle from δ_E to δ_F. The output of the generator is less than its input and the input to the motor is also less than its output. Thus the generator tends to accelerate whereas the motor gets decelerated;

Both these effects increases further the angle by which the generator leads the motor. Any displacement beyond point 'E', therefore results in the system pulling out of step.

However, we note that if the displacement from point 'E' was in an opposite direction, ~~the~~ (i.e. towards point 'D'), the solution will represent a stable operation. Hence, it can therefore be said that the point 'E' is a critical point in the system oscillation for the given internal voltages, reactances and power system can oscillate on either side of 'δ_A' upto 'δ_E', beyond 'δ_E' the system losses synchronism.

Assuming no losses, the system will be inherently stable for all power angles less than 90°, whereas beyond this angle, it will be unstable operation.

Thus, Steady State stability limit (P_{max}) under steady operation occurs at $\delta = 90°$ and is equal to $\frac{E_g \cdot E_m}{X}$ where E_g, E_m are voltages at generator and motor and 'x' is reactance of line.

Methods to Improve steady state Stability

(i) Reducing the reactance between the stations which can be made possible by adding machines or lines in parallel or by using machines of lower inherent impedance.

(ii) Optimum Conditions of $X = \sqrt{3} R$ for maximum power transfer is approached by using Series Capacitors for over head lines and series reactors for under ground cables

(iii) Higher excitation voltages

(iv) Quick response excitation system.

UNIT - ~~VI~~

UNIT-V

Power System Transient Stability Analysis

~~Swing~~ Swing Equation - Derivation

Under normal operations, the relative position of the rotor axis and the stator magnetic field axis is fixed.

The angle between the two axes is known as "load angle" or "torque angle" denoted by 'δ'. It depends upon the loading of the machine.

Larger the loading, larger is the value of the torque angle 'δ'. If some load is added (or) removed from the shaft of the synchronous machine, the rotor will decelerate (or) accelerate respectively with respect to the synchronously rotating stator field and a relative motion begins. It is said that the rotor is "swinging" with resp to stator field.

The equation describing the relative motion of the rotor with respect to stator field as a function of time is known as "Swing equation".

If 'T_s' represents shaft torque and 'T_e' the electromagnetic torque and if these are assumed as positive for a generator the net torque causing acceleration is

$$T_a = T_s - T_e \quad \text{— (1)}$$

A similar relation holds good when expressed in terms of power. i.e.,

$$P_a = P_s - P_e \quad \text{—— (2)}$$

where 'P_a' is accelerating power.

∵ Sync. machine is a rotating body, the laws of me[...]

apply to this also.

we know that $P_a = T_a \cdot \omega = I \alpha \omega = M \alpha$ — ③

$\omega = \frac{2\pi n_s}{60}$

From eqn ③, $M = I\omega$, where 'M' is in J-sec/mech.rad

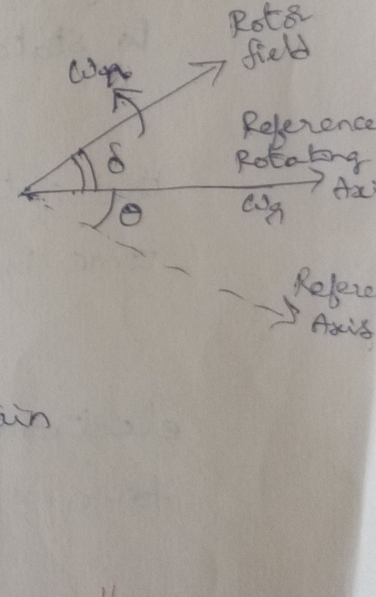
'M' is known as angular momentum and is expressed in terms of MJ-sec/elec.deg.

The acceleration 'α' can be expressed in terms of the angular position of the rotor as

$$\alpha = \frac{d^2\theta}{dt^2} \quad — ④$$

The angle 'θ' changes continuously w.r.t. time when a sudden change occurs in the system. The value of 'θ' is given by

$$\theta = \omega_r \cdot t + \delta \quad — ⑤$$



$$\Rightarrow \frac{d\theta}{dt} = \omega_r + \frac{d\delta}{dt} \quad — ⑥$$

and $$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \quad — ⑦$$

From equations ③, ④ and ⑦, we obtain

$$M \frac{d^2\delta}{dt^2} = P_a = P_s - P_e \quad — ⑧$$

Equation ⑧ is known as the "Swing equation". The angle 'δ' is the difference between the internal angle of the machine and the angle of the synchronously rotating reference axis.

Here 'P_s' is fixed and 'P_e' can be $\frac{V_1 V_2}{x} \sin\delta$ for a lossless system,

Then, $$M \frac{d^2\delta}{dt^2} = P_s - \frac{V_1 V_2}{x} \sin\delta = P_s - P_m \sin\delta \quad — ⑨$$

Equal Area Criterion

The equal area criterion is derived using the swing equation for a machine connected to an infinite bus.

The swing equation is given as

$$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_e$$

Multiplying both sides of the equation by $2\frac{d\delta}{dt}$ and integrating w.r.t. time, we get

$$\int 2M\frac{d^2\delta}{dt^2}\cdot\frac{d\delta}{dt}\cdot dt = \int 2(P_s - P_e)\frac{d\delta}{dt}\cdot dt$$

$$M\left(\frac{d\delta}{dt}\right)^2 = 2\int_{\delta_0}^{\delta}(P_s - P_e)\cdot d\delta$$

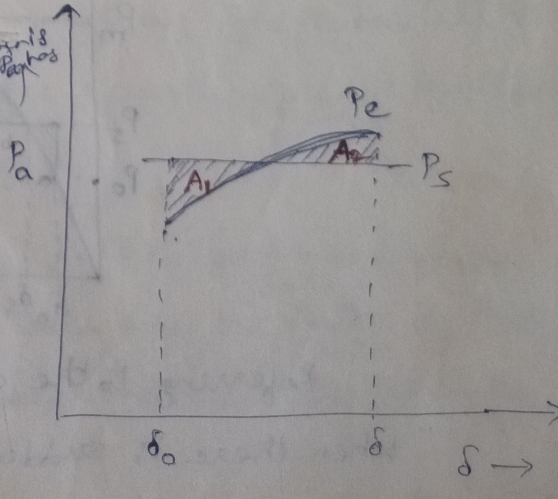
or $$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta}(P_s - P_e)\,d\delta + C} \quad -(10)$$

Where 'δ_0' is the initial torque angle before any disturbance occurs and at this time $d\delta/dt = 0$.

The angle 'δ' will stop changing and the machine will again be operating at synchronous speed after a disturbance when $d\delta/dt = 0$ (or) when

$$\int_{\delta_0}^{\delta}(P_s - P_e)\,d\delta = 0 = \int_{\delta_0}^{\delta} P_a\cdot d\delta \quad — (11)$$

This means that the area under the curve P_a should be zero which is possible only when P_a has both accelerating and decelerating powers, i.e. for a part of graph, $P_s > P_e$ and for the other $P_e > P_s$ as shown in the figure.



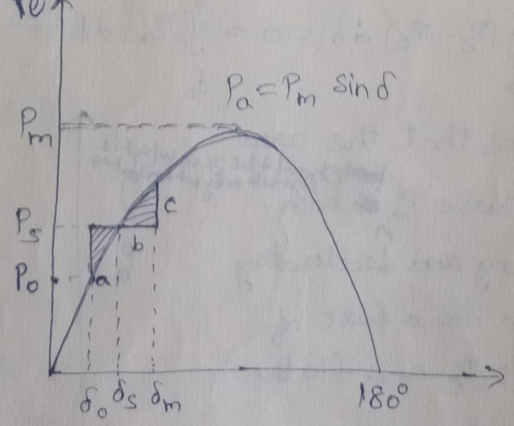
For a generator action $P_s > P_e$ for positive area and $P_e > P_s$ for negative area A_2 for stable operation. the name "equal area criterion".

The area 'A_1' represents the kinetic energy stored the rotor during acceleration and the area 'A_2' represents the kinetic energy given up by the rotor to the system. when it is all given up, the machine has returned to its original speed.

Sudden change of load - Equal Area Criterion

The following points are to be noted with regard to the change in torque angle whenever a disturbance occurs:

1. There is no change in torque angle when the speed of the rotor is the synchronous speed.
2. The angle increases in case of a motor if $P_s > P_e$ i.e. the mechanical output is more than the electrical input and the speed goes down.
3. The angle decreases if the speed is more than the synchronous speed.



Referring to the above figure, the operation of the motor when there is sudden increase in load can be explained as:

Initially, the motor is ~~at~~ operating with the torque angle δ_o with mechanical output P_o.

Now, let the load be increased to P_s. Momentarily, there is no change in angle δ corresponding to electrical input to the motor as $P_e < P_s$. Therefore, the motor decelerates as a result of this the torque angle increases and P_e starts increasing.

At point 'b', $P_e = P_s$ and therefore, decelerating force is zero but due to inertia of the rotor, the torque angle goes on increasing. The speed of the rotor beyond 'b' starts increasing. The speed is ~~minimum~~ minimum at 'b'.

When the speed equals the synchronous speed beyond point 'b', say at point 'c', the rotor angle stops increasing. But between 'c' and 'b', $P_e > P_s$. Therefore, the motor accelerates & the torque angle starts decreasing.

The speed goes on increasing till it reaches point 'b' where this time the speed is maximum & is more than the synchronous speed.

Between 'b' and 'a' $P_s > P_e$, therefore the rotor starts decelerating but the speed is more than sync. speed till it reaches the point 'a' where once again the speed is sync. speed and the angle stops decreasing.

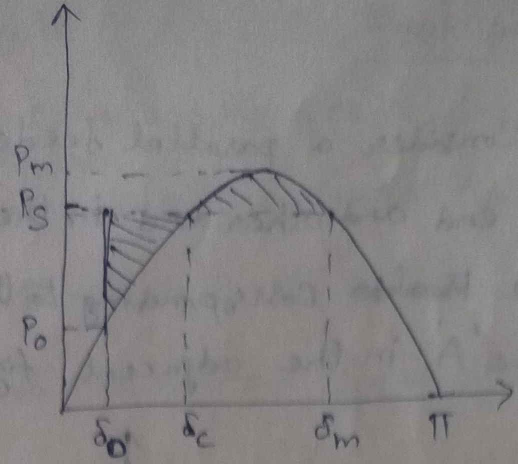
The speed at 'a' and 'c' is the sync. speed whereas at 'b', the speed is below sync. speed when the rotor oscillates from 'a' to 'b' and above sync. speed when the rotor oscillates from 'c' to 'b'.

The following table shows the changes in angle, speed, electric power input, mechanical power output and acceleration / deceleration as the machine oscillates between 'a' and 'c'. The machine will settle at 'b' after the oscillations are damped out.

Position	Torque angle	Motor Speed	Power	Acceleration (or) Deceleration
At pt. a	$\delta = \delta_0$	$\omega = \omega_n$	$P_e < P_s$	Deceleration
From a to b	Increasing	$\omega < \omega_n$	$P_e < P_s$	Deceleration
At pt. b	$\delta = \delta_s$	$\omega < \omega_n$	$P_e = P_s$	
From b to c	Increasing	$\omega < \omega_n$	$P_e > P_s$	Acceleration
At pt. c	$\delta = \delta_{max}$	$\omega = \omega_n$	$P_e > P_s$	
From c to b	δ decreasing	$\omega > \omega_n$ increasing	$P_e > P_s$	Acceleration
At pt. b	$\delta = \delta_s$	$\omega > \omega_n$ max	$P_e = P_s$	
From pt b to a	δ decreasing	$\omega > \omega_n$ decreasing	$P_e < P_s$	Deceleration
At pt. a	The cycle repeats itself			

We have seen that with power angle curve shown in above figure that if the load on the motor shaft is increased suddenly from P_0 to P_s, the system is stable.

For this system, let us find out the max. value of the "P_s" such that the system is critically stable. i.e. any attempt to increase P_s beyond this value the system becomes unstable.



Referring to above figure,

$$P_S = P_m \sin\delta_c = P_m \sin\delta_m$$

$$\therefore \delta_m = (\pi - \delta_c)$$

Here, 'δ_c' is known as "Critical torque angle".

For the two shaded areas to be equal, the following Condition should be satisfied.

$$P_S(\delta_m - \delta_0) = \int_{\delta_0}^{\delta_m} P_m \sin\delta \, d\delta \quad \text{— (12)}$$

Also $P_S = P_m \sin\delta_m$. Substituting this in eqn (12),

$$P_m \sin\delta_m (\delta_m - \delta_0) = \int_{\delta_0}^{\delta_m} P_m \sin\delta \, d\delta$$

$$P_m \sin\delta_m (\delta_m - \delta_0) = P_m(\cos\delta_0 - \cos\delta_m) \quad \text{— (13)}$$

Here 'δ_m' is the only unknown which can be obtained and hence 'P_S' can be calculated.

Applications of Equal Area Criterion

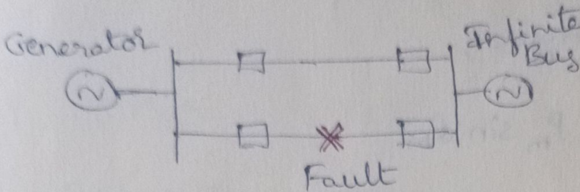
Equal area Criterion can be applied to two different systems of operation:

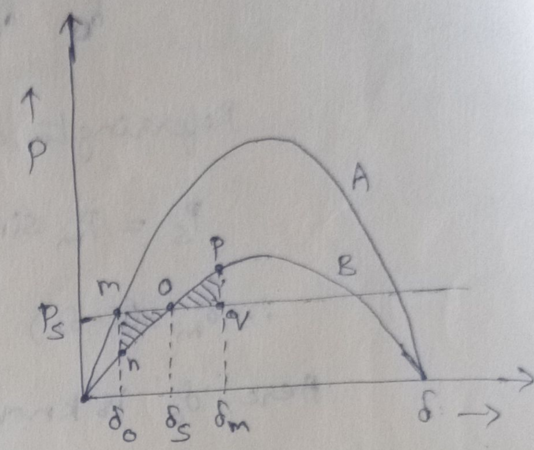
(i) Sustained line fault and

(ii) a line fault cleared after some time by simultaneous tripping of the breakers at both the ends.

(i) Sustained Line fault

Let us Consider a parallel feeder fed from the alternator on one end and other end is the infinite bus. The Power angle curve ~~is also~~ Corresponding to the healthy Condition is given by curve 'A' in the adjacent figure.





'P_s' is the input to the generator which is assumed Constant. Also the voltage behind transient reactance is assumed Constant. Curve 'B' represents power angle curve when a fault occurs on one of the two lines and the breakers operate instantaneously and simultaneously at both the ends on that line. As a result, the equivalent impedance between the busbars is increased and hence curve B will be lower than curve A.

Corresponding to input 'P_s' the torque ~~at~~ angle of the generator is 'δ_o' initially. Now, as soon as there is a fault and instantaneously it is cleared, the output of the generator goes down to point 'n' on curve 'B' and since input remains Constant which is higher than the output, the rotor accelerates and hence the torque angle increases and the operating point moves along curve 'B' towards 'o' from 'n'.

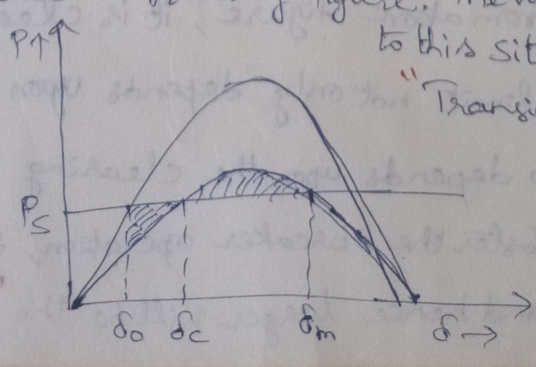
As it reaches 'o', the accelerating power 'P_a' becomes zero and the speed of the generator is more than the synchronous speed ~~infinite bus~~ and the speed continues to increase.

From 'o' to 'p' the rotor experiences deceleration but the speed is more than speed of ~~inf~~ infinite bus till it reaches point 'p' where the relative speed is zero and the torque angle ceases to increase.

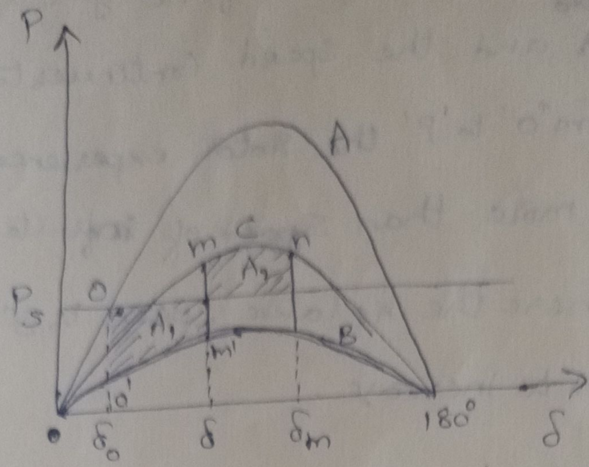
At point 'p', the output is more than the input to the generator and hence the rotor decelerates and the speed goes down relative to the infinite bus till it reaches point 'o' where the speed is minimum. This torque angle continues to decrease till it reaches point 'n' where again the speed is equal to the speed of infinite bus.

The cycle repeats itself if damping is not present. In practice, due to presence of damping, the rotor operates at point 'o' on curve 'B' and the torque angle is 'δ_s'.

To determine the transient stability limit for this case we should raise the input line 'P_s' such that the area below the line 'P_s' ~~on~~ and the curve 'B' and above the line 'P_s' and curve 'B' at the intersection of 'P_s' and 'B' are equal. This is as illustrated in following figure. The value of 'P_s' corresponding to this situation is known as "Transient Stability limit".



(ii) Fault cleared after some time



Curve 'A' in above figure represents the power angle curve corresponding to healthy condition of system.

Curve 'B' represents corresponding to fault on one of the two lines and fault allowed to exist for some time.

Curve 'C' corresponds to the situation when the faulted line is removed.

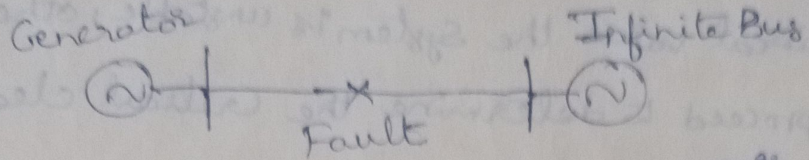
Initially for 'P_s' the torque angle is 'δ_0'. At the time of fault, the output of the generator becomes as at 'o''. Hence, the rotor accelerates and the rotor moves along the curve 'B' upto point 'm'' when the faulted line is removed and the operating point becomes 'm' on curve 'C' where the output is more than the input and the rotor decelerates till the speed becomes equal to the speed of the infinite bus and the torque angle ceases to increase at point 'n'.

From above figure, it is clear that the transient stability limit not only depends upon the type of disturbance but it also depends upon the clearing time of the breaker.

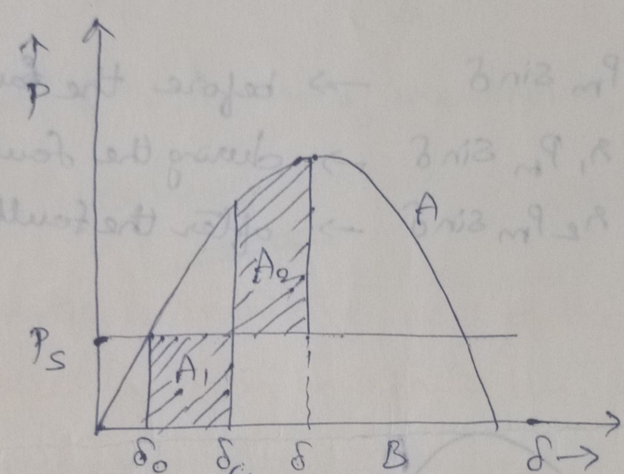
Faster the breaker operation, smaller will be the area 'A_1' and hence larger will be the transient stability limit.

4/10/16

Consider the another case where generator is connected to an infinite bus through a single feeder as shown below:



Say a three phase fault occurs on the line temporarily. The power angle curve will be as shown below for this case.



The power angle curve will correspond to the horizontal axis because power transferred is zero. If the breaker reclose after some time corresponding to clearing angle δ_c when the fault is cleared vanished, the output will be more than the input and hence the rotor decelerates. Finally, if the clearing angle 'δ_c' is such that $A_1 = A_2$, the system becomes stable.

Critical clearing Angle - Calculation

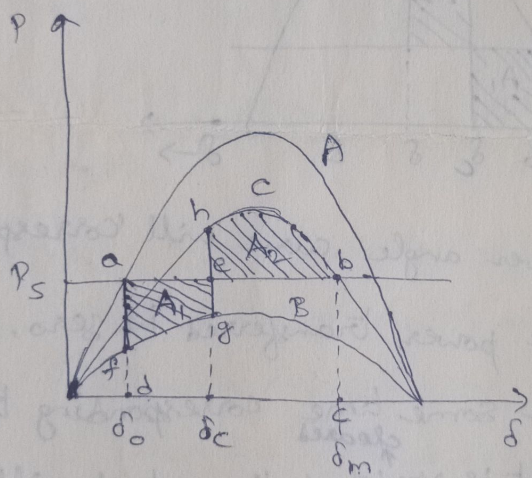
We see that for any given initial load there is a critical clearing angle. If the actual clearing angle is greater than the critical value the system is unstable; otherwise it is stable. So, we now proceed to determine the critical clearing angle for a given load.

Let the three power angle curves be represented as shown below.

$A = P_m \sin\delta \rightarrow$ before the fault

$B = r_1 P_m \sin\delta \rightarrow$ during the fault

$C = r_2 P_m \sin\delta \rightarrow$ after the fault



For transient stability limit, the two areas $A_1 = A_2$ (or) equivalently area under the curve abcd should be equal to the area under the curve dfghbc i.e.,

$$(\delta_m - \delta_0) P_s = \int_{\delta_0}^{\delta_c} r_1 P_m \sin\delta \, d\delta + \int_{\delta_c}^{\delta_m} r_2 P_m \sin\delta \, d\delta$$

$$= r_1 P_m [\cos\delta_0 - \cos\delta_c] + r_2 P_m (\cos\delta_c - \cos\delta_m)$$

Now substituting $P_e = P_m \sin\delta_0$

$$(\delta_m - \delta_0) P_m \sin\delta_0 = r_1 P_m (\cos\delta_0 - \cos\delta_m) + r_2 P_m (\cos\delta_c - \cos\delta_m)$$

$$(\delta_m - \delta_0)\sin\delta_0 = (x_2 - x_1)\cos\delta_c + x_1\cos\delta_0 - x_2\cos\delta_m$$

$$\therefore \cos\delta_c = \frac{(\delta_m - \delta_0)\sin\delta_0 - x_1\cos\delta_0 + x_2\cos\delta_m}{(x_2 - x_1)}$$

Now from the curves,

$$P_s = P_m \sin\delta_0 = x_2 P_m \sin\delta_m = x_2 P_m \sin(\pi - \delta_m)$$

(or) $\sin\delta_0 = x_2 \sin(\pi - \delta_m)$

(or) $\delta_m = \pi - \sin^{-1}\left(\frac{\sin\delta_0}{x_2}\right)$.

Thus, if x_1, x_2 and δ_0 are known, the critical clearing angle 'δ_c' can be obtained.

Ex:- (1) A motor is receiving 25% of the power that it is capable of receiving from an infinite bus. If the load on the motor is doubled, calculate the maximum value of δ during the swinging of the rotor around its new equilibrium position.

Sol:-

$\sin\delta_0 = 0.25$ $\quad \therefore \delta_0 = 14.48°$

$\sin\delta_c = 0.5$ $\quad \delta_c = 30°$.

$\delta_m = ?$

$$(\delta_m - \delta_0)\,0.5 = \int_{\delta_0}^{\delta_m} \sin\delta \, d\delta$$

$$0.5(\delta_m - \underbrace{14.48°}_{0.25272\ rad}) = (\cos\delta_0 - \cos\delta_m)$$

$$0.5\,\delta_m = \cos\delta_0 - \cos\delta_m + 0.1263$$

$$0.5\,\delta_m + \cos\delta_m = 0.96823 + 0.1263 = 1.094535$$

For solution of this equation we make guess for δ_m such that δ_m should be greater than $30°$ (as $\delta_c = 30°$). After some trials 'δ_m' is found to be $45°$.

Ex:- (2) A 50Hz generator is delivering 50% of the power it is capable of delivering through a transmission line to an infinite bus. A fault occurs that increases the reactance between the generator and the infinite bus to 500% of the value before the fault. When the fault is isolated, the maximum power that can be delivered is 75% of the original maximum value. Determine the critical clearing angle for the condition described.

Sol:- Let 'P_m' be the maximum power that can be delivered.

$$P_m \sin\delta_0 = 0.5\, P_m \quad (\text{or}) \quad \delta_0 = 30^\circ$$

During fault the reactance is 500% of the value before the fault. ~~When the fault is isolated the maximum power that can be delivered is 75~~

$$\therefore \; r_1 = 0.2 \quad \text{and} \quad r_2 = 0.75 \text{ (given)}$$

The critical clearing angle δ_c is given by

$$\delta_c = \cos^{-1}\left[\frac{(P_s/P_m)(\delta_m - \delta_0) + r_2\cos\delta_m - r_1\cos\delta_0}{r_2 - r_1}\right]$$

$$0.5\,P_m = 0.75\,P_m \sin\delta_m \Rightarrow \delta_m = 41.8^\circ \text{ (or) } 138.2^\circ$$

$$(\text{or}) \quad \delta_m = 2.412 \text{ radians}$$

Substituting these values in the above expression, we get

$$\delta_c = \cos^{-1}\left[\frac{0.5(2.412 - 0.5236) - 0.75\times 0.74547 - 0.2\times 0.866}{0.55}\right]$$

$$= \cos^{-1}(0.3836)$$

$$\boxed{\delta_c = 67.44^\circ}$$

Methods to Improve Transient Stability

The methods adopted to improve the transient stability in a system are:

(i) Use of high inertia machines

(ii) use of high speed governers which can quickly adjust generator input to load.

(iii) use of quick acting voltage regulators.

(iv) Excitation system is designed to give close voltage regulation under transient conditions.

(v) Reducing the severity of faults made possible by protecting against lightning, use of quick acting and autoreclosing circuit breakers, use of relays having a small time of operation.

(vi) By the use of high neutral grounding impedance.

(vii) Reduction of transfer reactance.

Some of the recent methods to improve stability are:

(i) HVDC Links

(ii) Breaking Resistors

(iii) By-passing valving

(iv) Full load Rejection Technique